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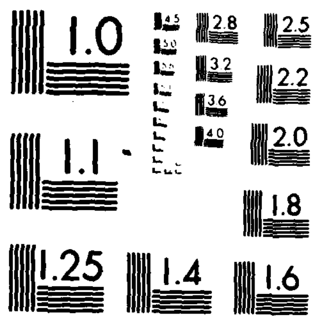

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The AF has experienced inventory support problems apparently because of Leadtime Variability. This study was to find out whether Leadtime Variability had a significant influence on safety stocks. The researcher fitted different distributions to Lead time data of High-Intensity items and found the data to fit the laplace distribution. Previous work ~~of Presutti and Tropp (1970)~~ provide an optimization model for the distribution. Use of this model can raise the average service level significantly. Cost of conversion for AFLC is → next page

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estimated at a onetime expense of $150 million and annual operating cost increase of $50 million.

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Phase III (item no. 0004): Air Force Contract F 33615-79-C-5143

LEADTIME VARIABILITY IN INVENTORY REQUIREMENT PROJECTIONS: FINAL RECHNICAL REPORT AND SUMMARY

Submitted to:

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June 30, 1980

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SUMMARY

This is the Phase III of AF Contract F 33615-79-C-5143: Leadtime Variability in Inventory Requirements Projections -- Final Technical Report and Summary. The report summarizes the first two technical reports and the results of the cost benefit analysis. The study consisted of three phases, and this is the third phase. The purpose of the study was to find out whether leadtime variability had a significant influence on stock safety levels. We found that, indeed, it has.

Currently, AFLC revises its leadtime according to the following model:

$$\hat{L}_t = L_{t-1} \quad ,$$

where $\hat{L}_t$ is the estimate of the required leadtime during period t and L_{t-1} is the actual leadtime observed during the preceding period. This model is inappropriate because it implicitly assumes that leadtime for the next-period-ahead is deterministic, whereas it is stochastic. Cohen, in a similar study for the U. S. Army, makes like observations (Cohen, 1979).

The first phase of the study was concerned with fitting distributions to leadtime data of high-intensity items. The results showed that procurement leadtime can be fitted very well by the lognormal, the gamma, the normal, or the Weibull. The gamma distribution has very often been postulated for the distribution of leadtime (Burgin, 1972, for example) and our results support this assumption.

The second phase was to be a simulation study and an analysis to examine the impact on inventory control levels of the fitted distributions of leadtime. The simulation model was to have been provided by AFLC, but in a February 5, 1980, meeting at WPAFB with Major Paul Gross of AFBRMC and Gloria

Picciano, Diann Lawson, and Carol Hawks of LORRA, it was agreed that it would be more beneficial to do independent simulations as well as look into several important issues not specified in the original contract. The oral agreement was embodied in a memo of understanding received on March 7, 1980. This memo is provided as Appendix A to this report.

Addressing the several issues raised by the memo of understanding, we first conjecture that the "skewed normal" distribution reported upon by Demmy (1979, p. IV-13) in his analysis of forecast error is likely produced by faulty analysis. We support this conjecture by simulation and by an analysis the reader will find in section 3. A good forecasting methodology should produce normally distributed forecast errors.

When leadtime and demand are stochastic, leadtime demand would be a compound distribution or a convolution. We knew the distribution of leadtime for the items in the sample, but we did not have the corresponding distributions for demand. Consequently, we assumed that daily demand at a depot is Poisson-distributed. We then obtained via simulation the compound distribution of leadtime demand for a few representative items. And we succeeded in fitting the right tails of these simulated leadtime distributions to the Laplace. This result is important because it verifies an important assumption in the work of Presutti and Trepp (1970). With Laplace leadtime demand, Presutti and Trepp have already worked out the optimization models. We also found that the normal distribution is a good approximation to the distribution of leadtime, but that the Laplace is better. Optimization techniques concerning the normal have been extensively worked out (see Brown, 1967). Section 5 of this report deals at some length with the distribution of leadtime demand.

Section 6 presents results of a cost-benefit analysis. Using a scenario at WR-ALC with shortage factor λ = 660, requisition size, R = 4, and essentiality parameter, Z_i = 0.5, we simulate the current system versus the Presutti-Trepp model IV (1970, pp. 248-249) optimal system. We find that the current system is wanting, the present average service level being 79.6% versus an optimal level of 86.9%. To bring the system to optimality for our sample requires a one-time investment of about $750,000 in safety stocks and an increase of about $250,000 in annual operating costs. With an annual procurement of one billion dollars, our sample represents 0.5% of that procurement. Thus, if the sample is representative, AFLC would require a one-time investment of $150 million in safety stocks and an increase in annual operating cost of $50 million.

It is also evident that the present mix of stock levels is inappropriate, and that the policy of maintaining an aggregate safety level stock not exceeding a two-month supply is not sound. The reader will discern these details in section 6.

1. DESCRIPTION OF THE STUDY

In recent years, the Air Force Logistics Command (AFLC), with an inventory of economic order quantity (EOQ) of over two billion dollars, has encountered inventory support problem apparently because of leadtime variability. The purpose of this study is to determine whether leadtime variability has a significant impact on inventory support planning and control at AFLC.

Using Air Force leadtime data on high-intensity items, we performed the runs test for randomness (Siegel, pp. 52-56). The leadtime data were random except for a few instances of suspected "pencil-whipping." With random leadtime data, we could then fit statistical distributions to them. This was accomplished by means of the Kolmogorov-Smirnov (KS) goodness-of-fit test (Conover, 1971, pp. 293-298), fitting the data to the exponential, the gamma, the normal, the Weibull, and the lognormal. The results indicated that, in general, the lognormal, the gamma, the normal, and the Weibull could be postulated as good fits to the leadtime data.

The impact of leadtime variability on stock levels can be known exactly if we identify the distribution of leadtime demand. To do that we need to know not only the distribution of leadtime but also of period demand. Since we had no demand data except item monthly demand rates, we assumed that daily demand is Poisson-distributed. Then with the best fit for leadtime and with demand Poisson, we simulated the distribution of leadtime demand for some representative items in the sample. The results indicated that the right half of the simulated distribution can be fitted very well to the Laplace. This is quite an important finding since AFLC would like to use

the Presutti-Trepp Model IV (1970, pp. 248-249) which assumes that leadtime demand is Laplace-distributed.

The final stage of the study was to conduct a cost-benefit analysis of current and proposed stock levels. This was accomplished with the aid of data received from LORRA on May 23, 1980. With that data and using Changes 2 and 3 of AFLC Regulations 57-6 (Department of the Air Force, 29 December 1978 and 22 June 1979), we were able to describe the current system. And by using the Presutti-Trepp Model IV, we were also able to generate a proposed optimal system.

2. SUMMARY OF THE FIRST TWO TECHNICAL REPORTS

According to the contract, the first technical report (Phase I) was to be delivered on December 31, 1979, the second (Phase II) on March 31, 1980. The Phase I report was to address the statistical distributions of leadtime, the Phase II report the AFLC simulation model. But because leadtime data were obtained in two subsets of sizes n = 16 and n = 46, the first in mid-October 1979 and the second toward the end of November 1979, and because of the memo of understanding (see Appendix A), the first two reports understandably did not conform precisely to the letter of the contract.

The Phase I report (27 pages) advanced the concept of leadtime demand as a compound distribution of period demand and leadtime. It also reported the results of goodness-of-fit tests on the first subset of sixteen items. Before doing the goodness-of-fit, the data were verified to be random.

The Phase II report (96 pages) completed the fitting of statistical distributions to the leadtime data, addressing the second subset of forty-six items. The Phase II report was made of seven technical appendices in order to facilitate future research and to serve as tutorials on the statistical distributions considered. Perhaps a very important Appendix in the Phase II report is the one that theoretically examines the influence of leadtime variability on buffer stocks and service levels, with examples provided on selected items.

3. THE DISTRIBUTION OF FORECAST ERRORS

In his analysis of forecast errors, Demmy (1979) arrived at distributions that are very much skewed to the right, distributions called "skewed normal" by LORRA. The purpose of this section is to show that when there is a mix dominated by declining demands for the many and sharply increasing demands for the few, then the method used by Demmy is very likely to produce distributions of forecast errors that are skewed to the right. We go on to validate our assertion by means of a set of experiments. But first we must say that data must be stationary and also random, that is uncorrelated, before distributions are fitted to them. This was evidently not in the case with Demmy's forecast errors. We concentrate on his Table IV-2 (Demmy, 1979, p. IV-13). The distributions he produced were based on approximately 22,500 items. For each of these items, he calculated a simple average and the MAD of the demands for the first eight quarters, that is, FY 71-72 (Demmy, 1979, pp. III-1 to III-3 and III-9 to III-11). The simple average became the forecast for each quarter in FY 73-75. Thus the forecast errors of quarters 9, 10, . . . , 20 were the actual demands for the quarter in question minus the simple average for the first eight quarters. Demmy then divided this forecast error by the historical MAD for that item in order to obtain standardized scores and in order to be able to aggregate the 22,500 odd forecast errors for each period. He called these standardized scores Z9, Z10, . . . , Z20.

If the demands for these items were stationary, then the distribution of forecast errors would have been symmetric if not normal. But these demands were not stationary, and it seems that most were declining with time. Consequently, the majority of the forecast errors were negative,

which explains the negative bias in forecast errors that Demmy obtained (Demmy, 1979, p. III-16). With declining demands it is obvious that the absolute value of the bias would increase with time: this is precisely what Demmy reports.

In summary we conjecture that tne "skewed normal" distributions obtained by Demmy are merely a result of wrong statistical analysis. Any forecasting method worth its salt should yield normally distributed forecast errors. Had Demmy accounted for the trends in demand, his forecast errors would have been normally distributed; and to validate our assertion, we perform a set of experiments.

A Set of Experiments

Eighty items are considered in the set of experiments. The demand function of every item can be described by the following expression:

$$D(t) = D(t-1) + C + X \,, \tag{3.1}$$

where $D(t)$ = demand in time period t,

C = a constant,

and X = a normally distributed random variable with zero mean and std. dev. equal to σ_X.

Each item belongs to one of the four classes described below:

Class I : C is negative.

Class II : C is zero.

Class III : C is positive.

Class IV : C is positive, but has a value of C' during the first eight time periods, and a value of C" during the next twelve time periods. In addition C" is greater than C'.

Each item has a unique combination of C and σ_X. For each item twenty demand values were generated by a mechanism consistent with (3.1).

Class I items show declining demand with a linear trend. Class II items exhibit no trend, whereas Class III items show increasing demand again with a linear trend. Class IV items also exhibit increasing demand but the trend changes abruptly after the eighth period.

<u>Forecasting Technique 1: Trend not removed</u>. The average demand in the first eight periods was the forecast for each of the next twelve periods. The standardized forecast error Z(t) was computed as follows:

$$Z(t) = \frac{D(t) - F(t)}{MAD} , \qquad t = 9, 10, \ldots, 20,$$

where $F(t)$ = forecast for period t $= \dfrac{\sum_{t=1}^{8} D(t)}{8}$,

and MAD = mean absolute deviation of demand during the first eight periods.

<u>Forecasting Technique 2: Trend removed</u>. The first difference S(t) = D(t) - D(t-1), was computed for the first eight periods. Values of D(0) were generated by the appropriate mechanism. The average value of S(t) in the first eight periods was a forecast of S(t) in each of the next twelve periods. Let the forecast of S(t) be FS(t), and the forecast of demand be SF(t).

$$\text{Then } FS(t) = \frac{S(1) + S(2) + \ldots + S(8)}{8} , \quad t = 9, 10, \ldots, 20;$$

$$\text{and } SF(t) = D(t-1) + FS(t) , \quad t = 9, 10. \ldots, 20 .$$

The standardized error SZ(t) is given by

$$SZ(t) = \frac{D(t) - SF(t)}{MAD} .$$

Experiments and Results -- Sample: Type A. Type A samples consisted of items from Classes I, II, and III. Each sample had eighty items. When the proportion of Class I items in the sample was higher than that of Class III, the distribution of Z(t) was found to be skewed to the right with a negative mean. For higher values of t, the mean shifted further away from zero. The distribution of SZ(t), however, was symmetric with a mean very nearly equal to zero.

As more and more Class III items were substituted for Class I items, the distribution of Z(t) became increasingly symmetric. When the proportion of Class III items exceeded that of Class I items, the distribution of Z(t) became skewed to the left with a positive mean.

The distribution of SZ(t) remained symmetric in spite of changes in the mix of items.

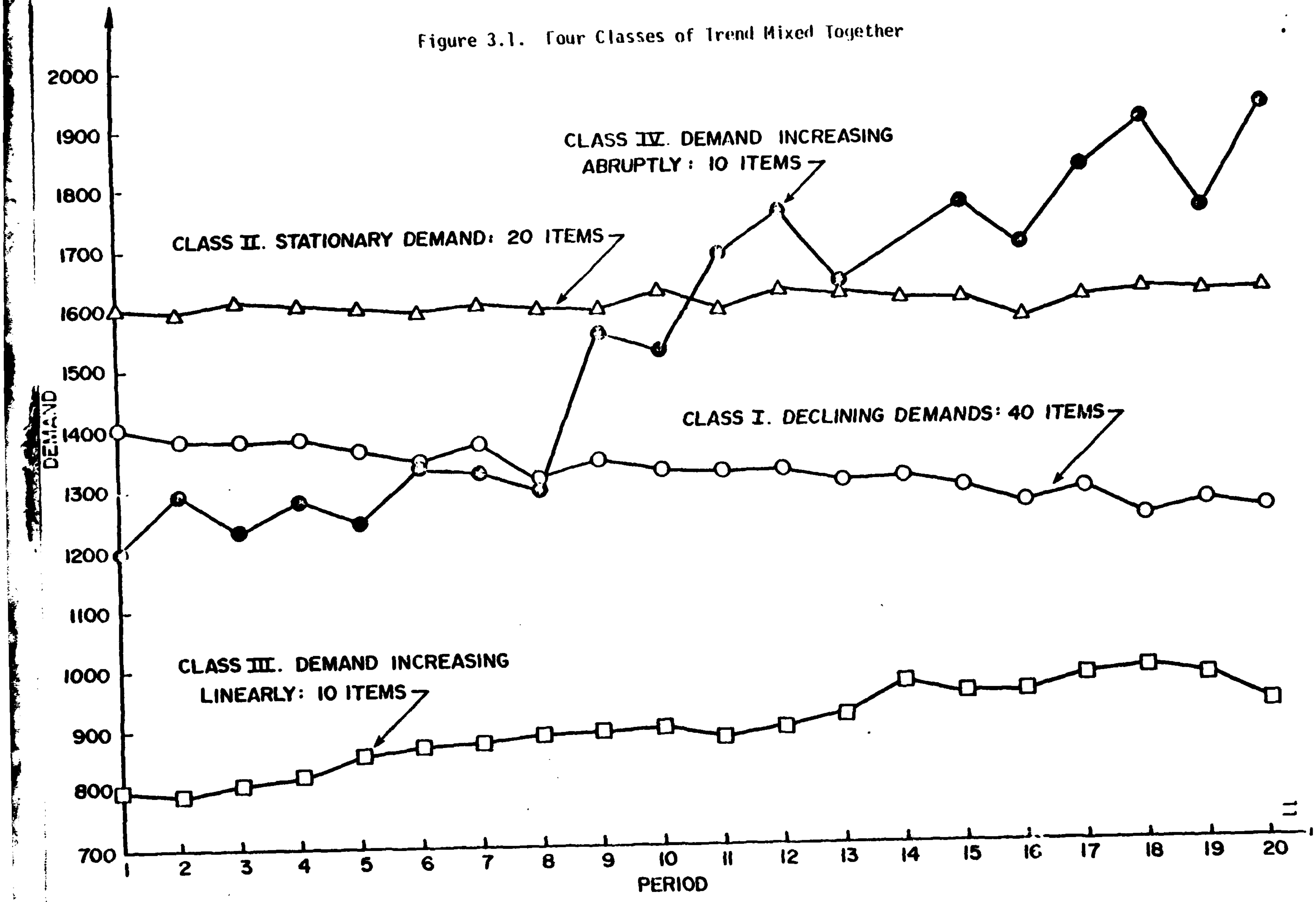
Experiments and Results -- Sample: Type B. Type B samples consisted of items from all four classes. A typical sample had the following composition:

40	Class I items
20	Class II items
10	Class III items
10	Class IV items.

An example of these classes of trend is given in Figure 3.1.

For this sample the Z(t) distribution was skewed to the right with a long right tail. This may be immediately seen in Figure 3.2; and for comparison we reproduce in Figure 3.3 Demmy's Z9 distribution. The tail vanished when Class IV items were dropped, though the distribution remained skewed to the right. The SZ(t) distribution was also found to have a long

Figure 3.1. Four Classes of Trend Mixed Together



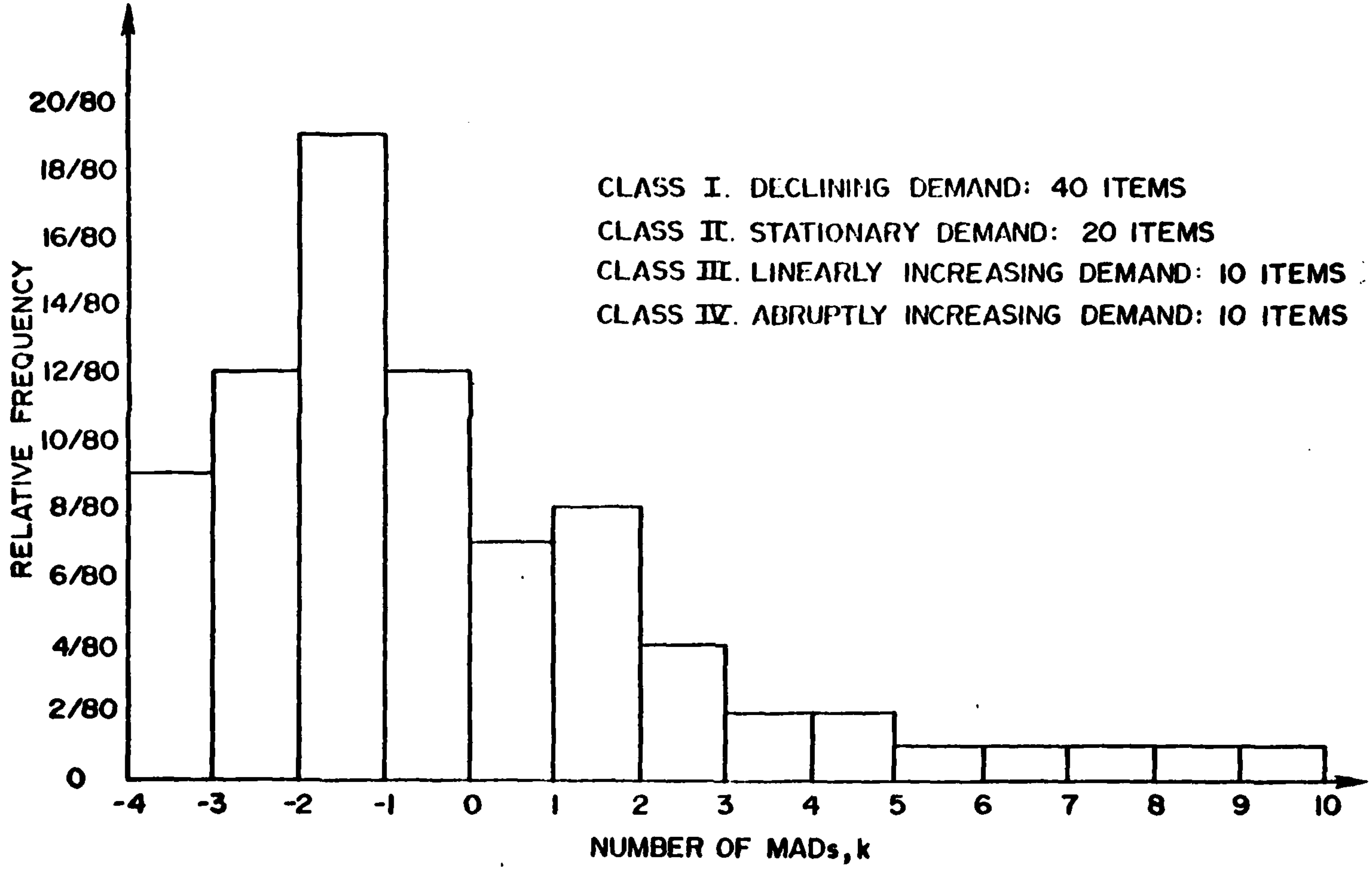


Figure 3.2. The Distribution of Forecast Errors, Z9, *for the Eighty Hypothetical* Items when the Data are Not Detrended.

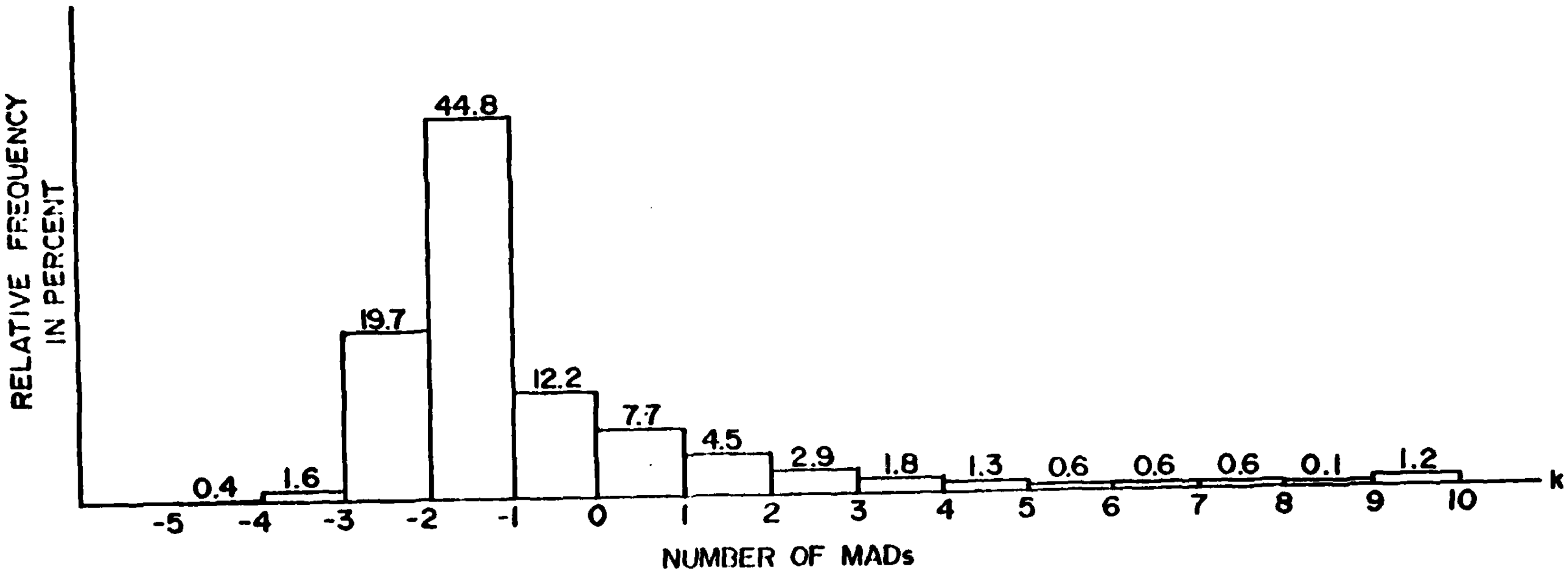


Figure 3.3. The Distribution of Forecast Errors, Z9, for OC-ALC Items with SMGC = 2 (The distribution is based on approximately 22,500 items)

Source: Demmy (1979, p. IV-13)

right tail. With the elimination of Class IV items, the tail disappeared and the distribution became symmetric around a mean that was very close to zero, as can be seen in Figure 3.4.

In conclusion, the assertion by LORRA that the distribution of forecast errors is highly skewed to the right does not seem valid. It is contrary to theory, and is most likely based on undue confidence in a very faulty forecast method.

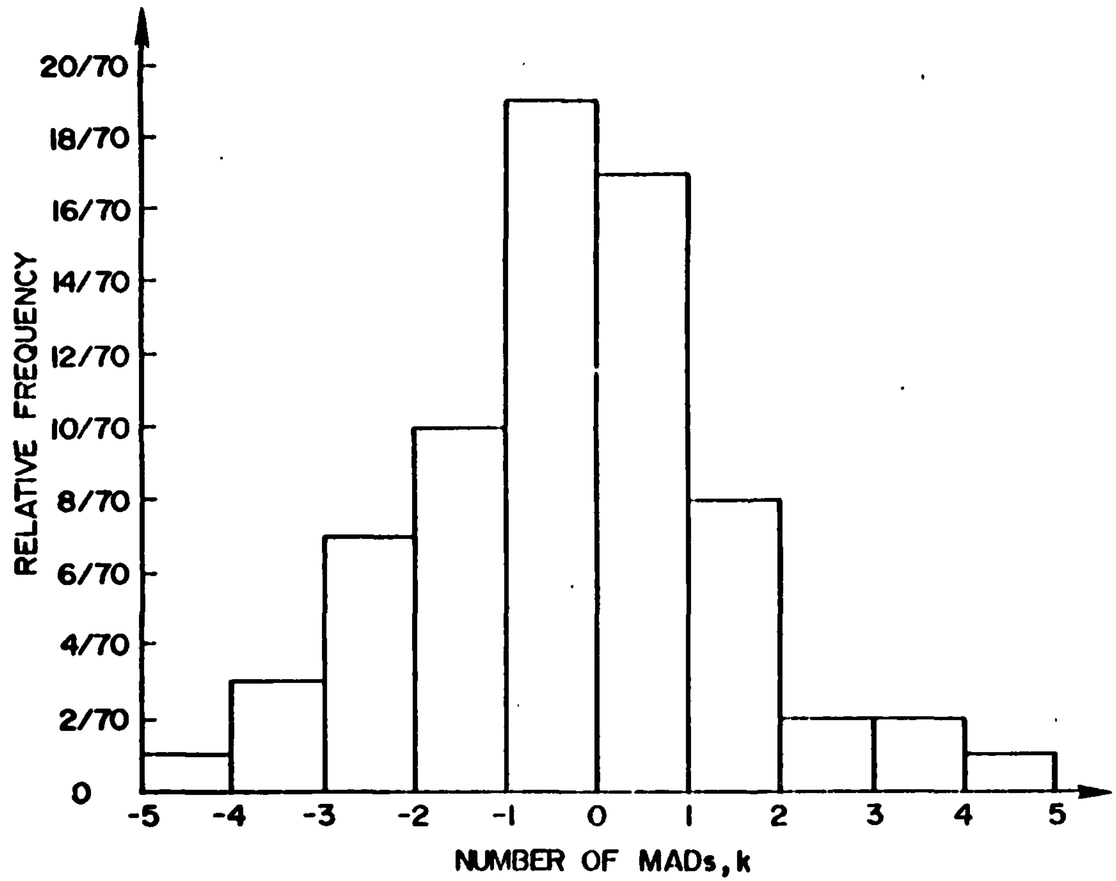


Figure 3.4. The Distribution of Forecast Errors, Z9, for the Same Items when the Linear Trends are Taken Out. (Class 4 is not considered because the pattern of the first eight periods changes abruptly)

4. ABC ANALYSIS

Because of the large number of items in the inventory, it is sometimes convenient to classify them according to the annual dollars of usage; and even though we are dealing only with high intensity items, it would still be useful to classify these. For the fifty-six items in the sample, we find that the total annual usage is $4,868,000, comprising about 155,000 units of demand. Of these 9.78% account for 54.34% of the annual dollars. We call these Class A items because they should receive the most attention. We similarly see that 25.64% of the number of units account for only 9.58% of the annual dollars. We call these Class C items because managing them can be kept as simple as possible. Between A and C, we have Class B items. For our sample, these comprise 64.58% of the number of units but account for 36.08% of the annual dollars. These results are taken from Table 4.1, and although arbitrary, the classification is consistent with the ABC classification scheme advocated in the literature. See, for example, Brown (1967, pp. 23-24) and Peterson and Silver (1979, pp. 71-73). The ABC classification is also consistent with the Supply Management Grouping Code (SMGC), even though the latter does not appear to be as useful. If we inspect Table 4.1, for example, we see that SMGC code M accounts for 79.46% of the annual dollars but 32.51% of the annual units; SMGC Code P accounts for 20.38% of the annual dollars but 66.66% of the annual units; and SMGC Code T accounts for 0.16% of the annual dollars and 0.83% of the annual units. In terms of the span of control, it should be clear that an ABC classification is superior to the SMGC. Furthermore, it would be easy to do tradeoffs between inventory investment and service level using ABC curves as Herron (1976) shows.

TABLE 4.1. ABC CLASSIFICATION FOR 1980 DATA

Item no.	FSN	Annual $'s	% of Annual $'s	Cum % $'s	Annual Units of Demand	% Units	Cum % Units	
37	1005007879802	1922543.17	.3949	.3949	7811.5	.0504	.0504	A items
15	5895001167508	438165.09	.0900	.4849	5583.0	.0360	.0864	9.58% units
21	3110006999957	284326.89	.0584	.5434	1772.0	.0114	.0978	54.34% $'s
33	1095004335657	170607.03	.0350	.5784	4872.5	.0314	.1292	
7	5960004009106	149757.20	.0308	.6092	4630.0	.0299	.1591	
11	5831008803563	144364.62	.0297	.6388	5737.0	.0370	.1960	
19	4210002727815	136725.60	.0281	.6669	5124.5	.0330	.2291	
23	1095009120256	127475.67	.0262	.6931	2191.6	.0141	.2432	
10	5895000894403	119067.36	.0245	.7176	643.6	.0041	.2474	
20	3110009429451	85625.10	.0176	.7351	1166.0	.0075	.2549	
24	1270000831981	78162.39	.0161	.7512	1091.0	.0070	.2619	
18	4310010339204	74917.01	.0154	.7666	8717.0	.0562	.3181	
25	1095010133419	68162.29	.0140	.7806	914.0	.0059	.3240	
26	1095008747369	68053.93	.0140	.7946	174.5	.0011	.3251	
43	1005006999923	49262.77	.0101	.8047	289.0	.0018	.3269	
47	1005003357318	47986.84	.0099	.8146	1534.0	.0099	.3368	
45	1005006999882	46970.71	.0096	.8242	12102.5	.0780	.4149	B items
27	1095009120243	45314.63	.0093	.8335	5669.0	.0366	.4514	64.58% units
53	1560009492087	44964.01	.0092	.8427	5373.5	.0346	.4861	36.08% $'s
42	1005007016793	44685.47	.0092	.8519	582.5	.0038	.4898	
32	1095004918437	39575.11	.0081	.8601	1072.6	.0069	.4967	
46	1005000511579	39527.64	.0081	.8682	3843.5	.0248	.5215	
12	5821002694508	39247.20	.0081	.8762	13461.0	.0868	.6083	
54	1560007803854	38237.76	.0079	.8841	916.0	.0059	.6142	
17	4440065749186	33322.18	.0068	.8909	18881.5	.1217	.7359	
8	5895009497160	32825.64	.0067	.8977	387.0	.0025	.7384	
35	1005008938230	31919.37	.0066	.9042	795.0	.0051	.7436	
2	5841000738392	29768.72	.0061	.9104	1782.0	.0115	.7551	
26	1095009553210	28330.64	.0058	.9162	660.0	.0043	.7593	
57	1560006117049	26955.40	.0055	.9217	371.5	.0024	.7617	
29	1095008F14744	26863.98	.0055	.9272	509.0	.0033	.7650	
40	1005007545267	26617.64	.0055	.9327	276.0	.0018	.7668	
50	7045008479020	26595.52	.0055	.9382	4487.0	.0289	.7957	
1	5826000146411	24803.10	.0051	.9433	381.0	.0025	.7982	
22	5365007855943	24507.00	.0050	.9483	2022.5	.0130	.8112	
48	1005000178809	24059.48	.0049	.9532	940.6	.0061	.8173	
59	1560004390187	23185.96	.0048	.9580	351.5	.0023	.8195	
51	3040006211345	20288.91	.0042	.9622	1546.9	.0100	.8295	
61	1440005727651	19964.06	.0041	.9663	2983.6	.0192	.8487	C items
55	1560006284732	18938.43	.0039	.9702	204.0	.0013	.8500	25.64% units
34	1095001111640	18432.57	.0038	.9739	1047.5	.0068	.8569	9.58% $'s
44	1005006999931	16809.29	.0035	.9774	860.5	.0055	.8623	
13	5821009906461	12294.49	.0025	.9799	2844.5	.0183	.8807	
30	1095005227703	11875.39	.0024	.9824	1638.8	.0106	.8913	
9	5835004451349	11098.84	.0023	.9846	301.0	.0019	.8932	
60	1560003101191	10449.05	.0021	.9868	530.5	.0034	.8966	
58	1560006099968	9685.45	.0020	.9888	1307.0	.0084	.9050	
36	1005007889718	8336.04	.0017	.9905	353.0	.0023	.9073	
31	1095005168069	8106.73	.0017	.9922	1456.6	.0096	.9167	
38	1005007755579	7920.95	.0016	.9938	1889.5	.0122	.9289	
39	1005007545293	6599.79	.0014	.9951	6773.0	.0437	.9726	
56	1560006220532	5644.80	.0012	.9963	820.6	.0053	.9779	
41	1005007545266	5215.91	.0011	.9974	699.5	.0045	.9824	
62	2530608269468	5119.80	.0011	.9984	1449.0	.0093	.9917	
52	1670007970253	4655.28	.0010	.9994	489.0	.0032	.9949	
3	5962004537739	3043.59	.0006	1.0000	797.5	.0051	1.0000	
		$4,867,959.50			155,098.2			

In Figure 4.1 is plotted the cumulative percentage of items along the horizontal axis versus the cumulative percentage of total dollar usage along the vertical axis. Figure 4.1 makes it more convenient to reclassify, if necessary.

According to Brown (1967, p. 23), the distribution of annual dollars of demand is lognormal. This is indeed the case for our data. The Lilliefors (L) test of lognormality yields a $D_{max} = 0.1097$, less than the critical statistic, $L_{0.95} = 0.886/\sqrt{56} = 0.1184$. Hence we accept the hypothesis of lognormality for the annual dollars of demand. The fit to lognormality is, in fact, quite good. (The parameters of the lognormal are $\mu = 10.369$ and $\sigma = 1.213$.)

In a similar vein, we also fit the number of annual units demanded to the lognormal distribution. This yields $D_{max} = 0.0890$, as compared to the critical $L_{0.95} = 0.1184$. Thus we also accept the hypothesis of lognormality for the number of items demanded. (The parameters of this lognormal are $\mu = 7.263$ and $\sigma = 1.157$.)

The 1980 Data

The data used to generate the Figure 4.1 were obtained from LORRA on May 23, 1980. In inspecting these data, it became clear that the sample of sixty-two we originally used has now been reduced to one of fifty-six items. This was because five of the items were no longer high-intensity, and one item was inadvertently duplicated in the original sample.

For the sake of future research and in order that our results may be replicated, we organize the data received on May 23, 1980 in Tables 4.2 and 4.3. These tables account for fifty-six items only; we have kept the original item numbers reflecting the listing in the Phase I and Phase II reports.

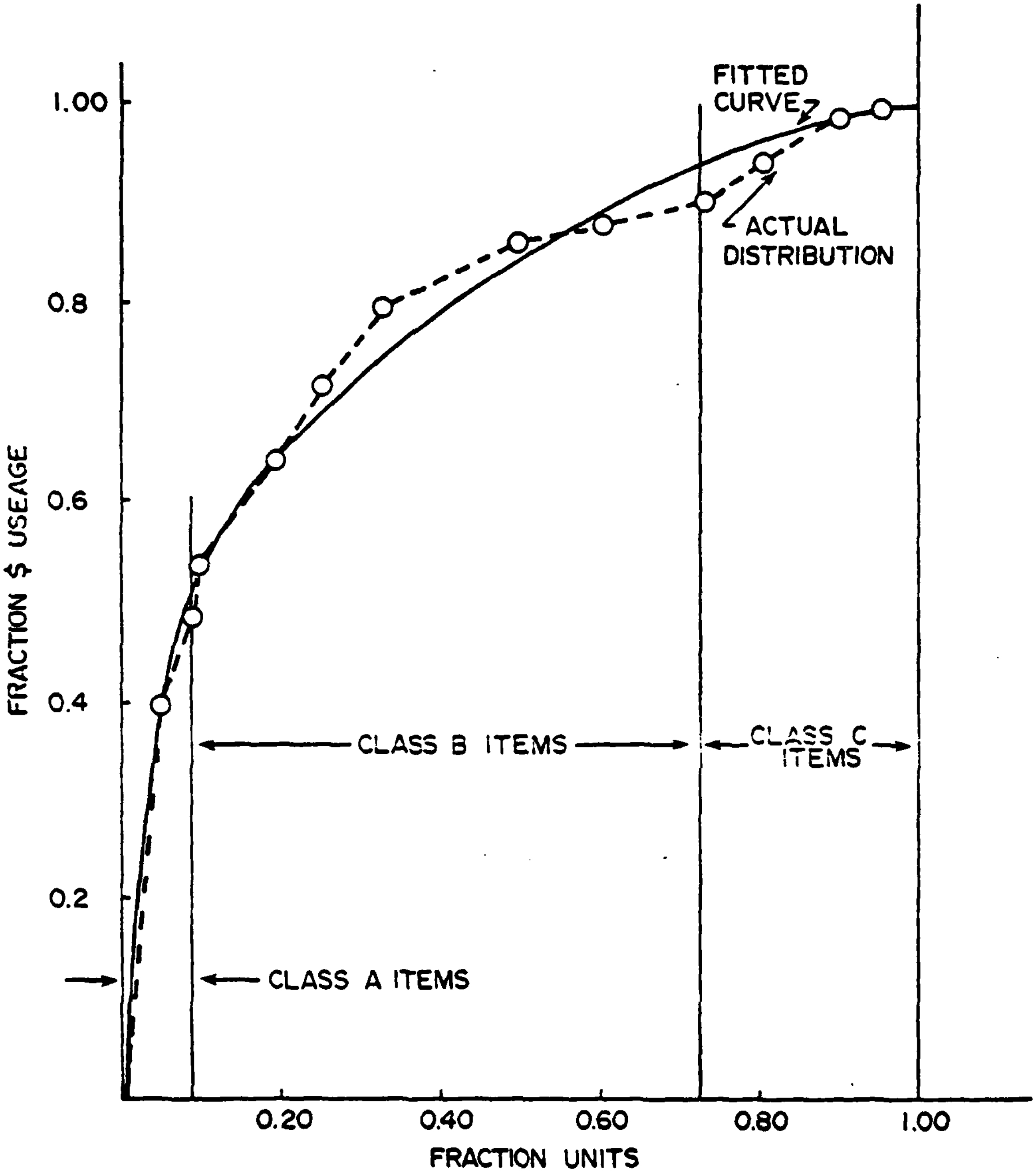


Figure 4.1. ABC Classification of the Sample Items--1980 Data

Source: Table 4.1

TABLE 4.2. DATA OF MAY 23, 1980 BY FSN

Item #	FSN	Monthly Demand	Quarterly MAD	Unit Price
[illegible]	[illegible]	650.96	433.60	3.08
[illegible]	[illegible]	465.25	923.06	7.08
[illegible]	[illegible]	147.67	128.75	27.08
[illegible]	[illegible]	406.04	371.94	9.64
[illegible]	[illegible]	385.83	215.00	10.64
[illegible]	[illegible]	478.08	201.25	2.93
[illegible]	[illegible]	427.04	109.88	8.72
[illegible]	[illegible]	182.63	124.40	2.38
[illegible]	[illegible]	53.63	73.07	41.36
[illegible]	[illegible]	47.17	85.38	5.66
[illegible]	[illegible]	90.92	102.25	7.26
37	1005007874602	726.42	666.50	220.55
[illegible]	1005007849718	75.17	82.26	9.12
[illegible]	1005008934230	14.54	19.88	182.94
[illegible]	[illegible]	23.33	14.75	65.84
[illegible]	[illegible]	127.83	68.25	111.22
[illegible]	[illegible]	1003.54	1082.13	3.27
[illegible]	[illegible]	472.42	298.32	1.43
[illegible]	[illegible]	447.79	344.53	2.21
[illegible]	[illegible]	48.54	39.72	46.12
[illegible]	[illegible]	49.35	135.66	53.45
[illegible]	[illegible]	320.29	351.98	11.79
[illegible]	[illegible]	1121.75	671.06	9.47
[illegible]	[illegible]	75.33	66.00	30.93
[illegible]	[illegible]	1573.46	525.47	3.61
[illegible]	[illegible]	32.25	19.25	201.97
[illegible]	[illegible]	71.71	37.16	23.20
[illegible]	[illegible]	157.46	83.47	5.53
[illegible]	[illegible]	25.08	24.50	77.04
[illegible]	[illegible]	504.42	329.75	1.43
[illegible]	[illegible]	55.35	37.09	32.95
[illegible]	[illegible]	53.29	45.65	6.07
[illegible]	[illegible]	120.75	74.75	13.07
[illegible]	[illegible]	29.42	28.07	105.31
[illegible]	[illegible]	121.38	53.86	30.87
[illegible]	[illegible]	40.75	136.13	9.52
[illegible]	[illegible]	56.25	70.19	6.44
[illegible]	[illegible]	55.46	30.03	25.44
[illegible]	[illegible]	44.21	22.36	535.94
[illegible]	[illegible]	148.50	172.58	48.05
[illegible]	[illegible]	55.00	54.50	207.16
[illegible]	[illegible]	33.75	26.13	201.65
[illegible]	[illegible]	42.33	32.25	65.60
[illegible]	[illegible]	104.92	70.25	18.75
[illegible]	[illegible]	23.00	25.50	142.20
[illegible]	[illegible]	373.92	540.75	2.74
[illegible]	[illegible]	31.75	14.55	65.10
[illegible]	[illegible]	155.54	55.13	71.38
[illegible]	[illegible]	75.38	53.88	31.65
[illegible]	[illegible]	29.29	29.04	338.76
[illegible]	[illegible]	128.91	119.13	283.25
[illegible]	[illegible]	245.53	240.40	3.72
[illegible]	[illegible]	17.00	17.75	160.91
[illegible]	[illegible]	135.57	114.75	91.36
[illegible]	[illegible]	237.04	219.55	1.07
[illegible]	[illegible]	87.29	115.06	25.39

Table 4.3. DATA OF MAY 23, 1980 BY ITEM NUMBER

Item #	FSN	Monthly Demand	Quarterly MAD	Unit Price
1	5826000146411	31.75	14.56	65.10
2	5841000738392	78.38	63.88	31.65
3	5962004537739	237.04	219.85	1.07
7	5960004009106	136.57	114.75	91.38
8	5895009497160	17.00	17.75	160.91
9	5895004451349	248.63	240.40	3.72
10	5895000894403	29.29	29.09	338.76
11	5831008903563	168.54	68.13	71.38
12	5821002694508	23.00	25.50	142.20
13	5821009906461	373.92	540.75	2.74
15	5895001167508	128.91	118.13	283.25
17	4440005749186	42.33	32.25	65.60
18	4310010339204	30.96	26.13	201.65
19	4210002727815	55.00	88.50	207.16
20	3110009429451	148.50	172.88	48.05
21	3110006999957	44.21	22.38	535.94
22	5365007655943	108.92	70.25	18.75
23	1095009120256	1121.75	671.06	9.47
24	1270000831981	32.25	19.25	201.97
25	1095010133419	1573.46	525.47	3.61
26	1095009553210	76.33	66.00	30.33
27	1095009120243	320.29	351.88	11.79
28	1095008747369	89.38	136.66	63.45
29	1095008614744	48.54	39.72	46.12
30	1095005227703	447.79	344.53	2.21
31	1095005168069	472.42	299.32	1.43
32	1095004918487	1008.54	1082.13	3.27
33	1095004335657	127.93	68.25	111.22
34	1095001111640	23.33	14.75	65.94
35	1005008938230	14.54	19.98	182.94
36	1005007889718	76.17	32.26	9.12
37	1005007979832	726.42	666.50	220.55
38	1005007755579	90.92	102.25	7.26
39	1005007545293	97.17	85.38	5.66
40	1005007545267	53.63	73.07	41.36
41	1005007545266	182.63	124.40	2.38
42	1005007916733	427.04	109.39	3.72
43	1005006999923	395.83	215.00	10.64
44	1005006999931	479.08	201.25	2.83
45	1005006999982	406.04	371.94	9.64
46	1005000511573	465.25	923.06	7.09
47	1005003357318	147.67	128.75	27.06
48	1005001178809	650.96	483.60	3.09
50	7045008479020	97.29	115.06	25.39
51	3040006211345	66.46	30.03	25.44
52	1670007970253	40.75	136.13	9.52
53	1560009492087	121.38	53.88	30.87
54	1560007803854	29.42	29.07	109.31
55	1560006284732	120.75	74.75	13.07
56	1560006220532	58.29	45.88	8.07
57	1560006117049	68.38	37.09	32.95
58	1560006099969	564.42	320.75	1.43
59	1560004330187	25.09	24.50	77.04
60	1560003101191	157.46	83.47	5.53
61	1440005727651	71.71	37.15	23.[illegible]
62	2530003263468	66.25	70.19	6.44

Table 4.2 lists the items by ascending order of stock number; Table 4.3 lists these items by ascending order of item number. Note in Table 4.3 that these items were deleted: item 4 (FSN 302101866582JH: Latch), 5 (FSN 1560005722616JH: Rod Assembly), 6 (FSN 5305001117850AB: Screw Spec), 14 (FSN 589500172114: Antenna), 16 (FSN 7045008479020), and 49 (FSN 4935006506352AB: Cable Assys - 2). Item 16 is in fact item 50 (FSN 7045008479020); the other items are no longer high-intensity.

The 1977-78 Data

We had, prior to obtaining the 1980 data, played detective with 1977-78 order quantities and unit prices originally provided with leadtime data. We thus generated the annual units demanded as well as the dollar values of demand for 1977-78. For the sake of comparison with 1980 data, we provide Table 4.4 and Figure 4.2. Table 4.4 is the analogue of Table 4.1, and Figure 4.2 is the analogue of Figure 4.1. In Table 4.4, we see that 5.68% of the top units account for 63.09% of the dollars; 56.8% of the intermediate units account for 34.79% of the dollars; and 37.52% of the bottom units account for only 2.12% of the dollars. In 1977-78, the sample accounted for an annual usage of $6,940,000 and comprised 399,528 units of demand. Thus, the demand for these items has fallen, on the average, between 1977 and 1980. An item-by-item comparison, using Tables 4.1 and 4.4, shows that a majority have declined in demand, but that for a few, demand has sharply increased. This is consistent with our conjectures in Section 3.

The distribution of annual dollars of demand in Table 4.4 is also lognormal. The Lilliefors (L) test of lognormality yields a D_{max} of 0.07017. This is much less than the critical statistic $L_{0.95} = 0.886/\sqrt{62} = 0.1125$. Hence we accept the hypothesis of lognormality, the parameters of the

Table 4.4. ABC CLASSIFICATION BASED ON 77-78 DEMANDS & PRICES

Annual Dollar Usage	% of Total $'s	Cumulative % $'s	Units of Demand	% Units	Cumulative % Units	Item no.	
[illegible]	[illegible]	[illegible]	[illegible]	.0264	[illegible]	[illegible]	Class A 5.7% units 63% $'s
[illegible]	.0790	[illegible]	1145.0000	.0029	[illegible]	21.0000	
[illegible]	[illegible]	[illegible]	[illegible]	.0035	[illegible]	10.0000	
[illegible]	[illegible]	[illegible]	[illegible]	.0008	[illegible]	22.0000	
[illegible]	.0273	[illegible]	2006.0000	.0075	.0551	11.0000	
[illegible]	.0260	[illegible]	[illegible]	.0017	[illegible]	[illegible]	
172593.2000	.0240	.5557	20645.0000	.0517	.1595	23.0000	Class B 57% units 35% $'s
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.1127	[illegible]	
[illegible]	.0200	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	.7176	[illegible]	[illegible]	.1519	[illegible]	
[illegible]	.0150	.7316	1700.0000	[illegible]	[illegible]	[illegible]	
[illegible]	.0138	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2005	20.0000	
[illegible]	[illegible]	[illegible]	1167.0000	[illegible]	.2036	7.0000	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	40.0000	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	.0116	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	.0111	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54.0000	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	29.0000	
[illegible]	[illegible]	[illegible]	[illegible]	.0010	[illegible]	[illegible]	
58174.0000	[illegible]	.8720	[illegible]	[illegible]	[illegible]	45.0000	
51907.4510	.0075	.8796	[illegible]	[illegible]	[illegible]	2.0000	
[illegible]	.0072	.8867	[illegible]	[illegible]	[illegible]	47.0000	
45670.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	32.0000	
44325.0000	[illegible]	[illegible]	1976.[illegible]	[illegible]	[illegible]	4.0000	
42244.9000	.0061	[illegible]	[illegible]	.0067	[illegible]	27.0000	
41535.0000	.0060	.9117	[illegible]	.0037	[illegible]	8.0000	
[illegible]	.0059	.9171	[illegible]	[illegible]	.3519	42.0000	
[illegible]	.0055	.9226	[illegible]	[illegible]	[illegible]	44.0000	
[illegible]	.0055	[illegible]	12020.[illegible]	.0201	[illegible]	25.0000	
[illegible]	.0054	[illegible]	[illegible]	[illegible]	[illegible]	49.0000	
[illegible]	.0051	[illegible]	[illegible]	[illegible]	[illegible]	19.0000	
[illegible]	.0049	[illegible]	[illegible]	[illegible]	.3990	[illegible]	
31362.0000	.0045	[illegible]	[illegible]	[illegible]	[illegible]	59.0000	
30734.0350	.0044	[illegible]	[illegible]	.0113	[illegible]	1.0000	
29934.2000	.0042	.9570	[illegible]	.0012	[illegible]	17.0000	
23946.8750	.0035	.9605	197.0000	.0005	[illegible]	12.0000	
22190.7400	.0034	.9639	821.0360	.0021	.4383	[illegible]	
21442.5900	.0031	.9670	309.0000	.0009	.4392	24.0000	
[illegible]	[illegible]	.9699	[illegible]	.0251	.4649	30.0000	
16653.6000	.0024	.9722	119.0000	.0015	.4663	26.0000	
16454.0790	.0024	.9746	60941.0000	.1524	.6188	50.0000	
14888.3500	.0021	.9768	[illegible]	.0017	.6205	51.0000	
14271.6000	.0021	.9788	1699.0000	.0063	.6268	52.0000	
13622.4800	.0020	.9808	4153.6000	[illegible]	.6352	9.0000	Class C 37.5% units 2% $'s
13555.0700	.0020	.9829	6846.5000	.0171	.6523	13.0000	
13312.8900	.0019	.9847	49307.0000	.1236	.7757	16.0000	
11477.0000	.0017	.9863	2300.0000	.0058	.7815	39.0000	
11386.7250	.0016	.9880	1414.5000	.0035	.7850	36.0000	
10932.0300	.0016	.9895	67537.0000	.1440	.9290	6.0000	
10613.7800	.0015	.9911	1643.0000	.0041	.9331	38.0000	
10225.7400	.0014	.9926	878.[illegible]	.0022	.9353	55.0000	
8666.8800	.0012	.9938	1770.0000	.0044	.9398	60.0000	
8362.2000	.0012	.9950	3982.0000	.0100	.9497	41.0000	
7987.2000	.0012	.9962	6240.0000	.0156	.9654	58.0000	
7073.0000	.0011	.9973	5950.0000	.0149	.9802	31.0000	
7472.7600	.0011	.9984	1035.5000	.0026	.9828	56.0000	
5786.7100	.0008	.9992	1017.0000	.0025	.9854	62.0000	
5547.0500	.0008	1.0000	5839.0000	.0146	1.0000	3.0000	

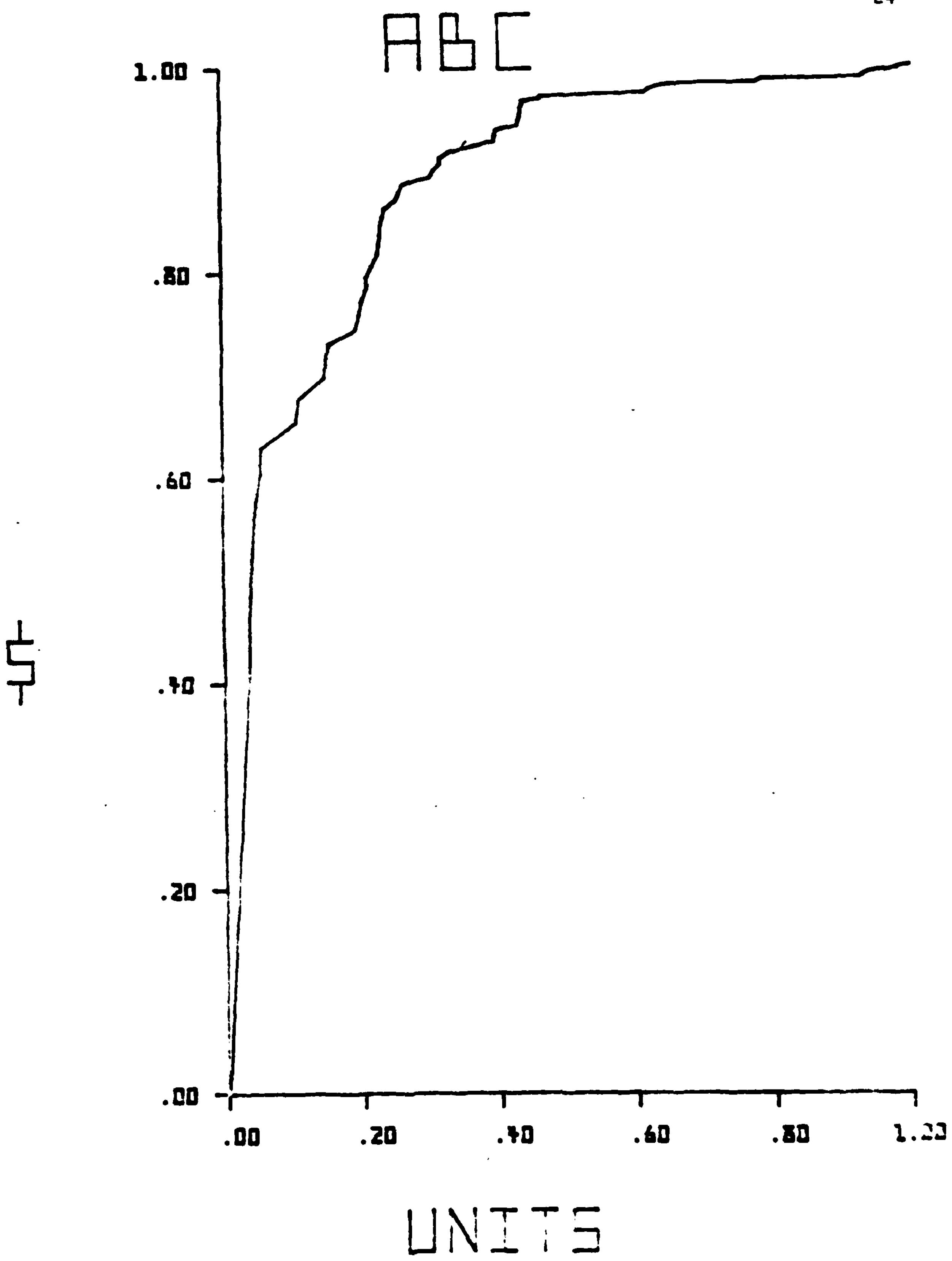


Figure 4.2. ABC Classification of the Sample Items--1977-73 Data

Source: Table 4.4

lognormal being μ = 10.595 and σ = 1.210. Furthermore, the number of annual units demanded is also lognormal with parameters μ = 7.717 and σ = 1.407. The L test yields a D_{max} of 0.1097, less than the critical $L_{0.95}$ = 0.1125.

It should be observed that the analysis of the 1977-78 imputed data is based on the original sample of size sixty-two which contains one redundant item (#16) and five items (#4, 5, 6, 14, and 49) that are no longer high-intensity. Since item 16 plays a very minor role, the effect of its presence is negligible.

5. THE DISTRIBUTION OF LEADTIME DEMAND

We may think of leadtime demand as a random sum of demands that are independently and identically distributed. Thus leadtime demand may be written as

$$X = D_1 + D_2 + \dots + D_i + \dots D_L, \; i = 1, 2, \dots, L \; , \qquad (5.1)$$

where D and L are random variables denoting demand and leadtime. Thus leadtime demand, X, may be thought of as a mixture, while leadtime, L, is the mixing distribution. More specifically, f(X) may be said to be a <u>compound distribution</u> with G(L) being the compounding distribution (Ord, 1972, pp. 64-66).

It can be shown (Drake, pp. 109-112) that for the structure (5.1)

$$E(X) = E(L) \,.\, E(D) \; , \qquad (5.2)$$

and

$$V^*(X) = E(L) \,.\, V(D) + [E(D)]^2 \,.\, V(L) \; , \qquad (5.3)$$

the star denoting the variance of leadtime demand with variable leadtime.

If, on the other hand, leadtime is constant at L, then E(X) would be as in (5.2) but

$$V(X) = E(L) \,.\, V(D) \; . \qquad (5.4)$$

We can immediately see the influence on safety stocks if we begin to consider the variability of leadtime. For the same safety factor, k, this increase would be in the ratio

$$\frac{\overset{*}{\sigma}(X)}{\sigma(X)} = \left(\frac{E(L)\ V(D) + [E(D)]^2\ V(L)}{E(L)\ V(D)}\right)^{1/2}$$

$$= \left(1 + \frac{VMR(L)}{VMR(D)} \cdot E(D)\right)^{1/2} \text{, where VMR denotes the variance to mean ratio} \quad (5.5)$$

$$\simeq \sigma(L)\left(\frac{E(D)}{E(L)}\right)^{1/2} \text{, for } VMR(D) = 1 \text{ and large } VMR(L) .$$

A similar analysis appears in Appendix G of the Phase II report (Hayya, March 31, 1980, pp. 88-94). An explicit treatment of the impact on safety stocks will be considered in the next section.

Theory of Compound Distributions

It would be easy if compound distributions were readily recognizable: sometimes they are, sometimes they are not. Hadley and Whitin (1963, p. 117) have shown that where the procurement leadtime is gamma-distributed with parameters α, β, and if a Poisson process with mean λt generates demands with units being demanded one at a time, then the distribution of leadtime demand is a negative binomial with parameters $\alpha + 1$, $\beta/(\beta + \lambda)$. Burgin (March 1972) has treated the case with demand normal and leadtime gamma. There is other work (for example: Sherbrooke, 1968; Ord, 1972; Bott, 1977), but the leadtime distributions are too complex for our present purposes.

In the absence of data on daily demands, the assumption of Poisson demands may be appropriate. Furthermore, the fitting of statistical distributions to leadtime data supports the notion that leadtime is gamma-distributed. Consequently, we can assume that leadtime demand is a negative binomial. Better yet, we can use simulation in order to see whether we can fit the distribution of leadtime demand for our items to the normal or the Laplace. Particularly, if we can fit the distribution of leadtime demand to the Laplace, we can take advantage of the models developed for the Air Force by Presutti and Trepp (June 1970)

Simulation Experiment 1: Poisson Demands; Best Fit for Leadtime--1977-78 Demands

We had no demand distribution data at the writing of this report; and until May 25, 1980, we had no 1980 daily demand data. Consequently we made the assumption that daily demand at the depot is Poisson-distributed, and, at the beginning, we estimated mean daily demand by averaging the quantities ordered over the two most recent years of data: 1977 and 1978. Then we estimated the annual dollars of demand. These ranged from $6,000 for item no. 3 (Integrated, FSN: 5962004537739) to $2,825,000 for item no. 37 (20 mm GunBa, FSN: 1005007879802). On the other hand, the standard deviation of leadtime demand, σ_i^*, ranged from 29.33 for item no. 12 (Antenna, FSN: 5821002694508) to 15,385.47 for item no. 23 (PAD-ASSY 1, FSN: 1095009120256). We could see by inspection that dollar value of demand and σ_i^* are hardly related. A calculation of the linear correlation coefficient yields $r = 0.256$, barely significant at the 0.05 level (the calculated value of the test statistic is $t(60) = 2.055$). It may, however, be possible to fit an exponential function to the two variables.

A cross-tabulation of annual dollars of demand versus σ_i^* produces the frequencies given in Figure 5.1. From the cross-tabulation in Figure 5.1, we choose five items for a simulation experiment. These are:

Item no.	Noun: FSN	σ_i^*	Annual $'s
23	Pad Assy-1: 1095009120256	Very high (15,385)	High ($173,000)
37	20 mm. Gun Ba: 1005007879802	High (7,423)	Extremely high ($2,825,000)
39	Guide Rolle: 1005007545293	Low (384)	Very low ($11,000)
50	Lead Tape: 7045008479020	Very high (10,530)	Very low ($16,000)
53	Adapter: 15600094987JH	Moderate (1346)	Moderate ($79,000)

Standard Deviation of Leadtime Demand, $\sigma_i^*(X)$	Annual $'s (in thousands): Very low 6 - 20	Low 20 - 50	Moderate 50 - 100	High 100 - 200	Very high 200 - 600	Extremely high >600	Totals
Very low (30-200)	//	~~////~~ ////	////	///			18
Low (200-800)	~~////~~ /// item #39	///	////		///		18
Moderate (800-2000)	~~////~~	///	//// item #50	/			13
High (2,000-10,000)	///	///	//	//		/ item #37	11
Very high (10,000-16,000)	1 item #50			1 item #23			2
Totals	19	18	14	7	3	1	62

Figure 5.1. A Two-Way Classification of Annual Dollars Versus the Standard Deviation of *Leadtime Demand*--1977-78 Data

The best fit for the leadtimes and corresponding parameters is given in Table 5.1. Table 5.2 gives the mean and variances of leadtime demand. With Poisson demands and with leadtimes as described in Table 5.1, we generate by simulation distributions of leadtime demand, using samples of size $n = 30$ and $n = 100$. We then fit these distributions to the normal and to the Laplace. The results are given in Table 5.3: we see in that table that we are not successful in fitting the normal or the Laplace for the entire distribution of leadtime demand; we are, however, quite successful in fitting the normal and the Laplace to the right half of the distribution of leadtime demand, that is, for $k > 0$.

Simulation Experiment 2: Poisson Demands; Best fit for Leadtime--1980 Demands

Since demands declined in general from 1977 to 1980, we verified the results of the previous simulation using 1980 demands. Using samples of size $n = 30$, we again simulate daily Poisson demands and the leadtime distributions given in Table 5.1.

The means and variances of leadtime demand, using 1980 data, are given in Table 5.4; it is the analogue of Table 5.2. The goodness-of-fit of the 1980 leadtime data (actually 1980 demands and 1975-78 leadtimes) to the normal and the Laplace are also verified as seen in Table 5.5. It is seen in Table 5.5 that the fit is better in the right tail of the distribution; the fit improves dramatically beyond $k = 1$. This is what we are looking for.

TABLE 5.1

BEST FIT FOR LEADTIME

Item no. FSN	Best Fit	Parameters
23: 1095009120256 (Pad Assy-1)	Normal	μ = 384.57; σ = 272.00
37: 1005007879802 (20 mm. Gun Ba)	Gamma	α = 6.09; β = 75.19
39: 1005007545293 (Guide Rolle)	Lognormal	μ = 4.90; σ = 0.41
50: 7045008479020 (Lead tape)	Gamma	α = 5.63; β = 26.60
53: 1560009492087JH (Adapter)	Gamma	α = 3.40; β = 91.74

TABLE 5.2

MEANS AND VARIANCES OF LEADTIME DEMAND: 1977-78 DEMAND DATA

Item No.	1977-80 Expected Daily Demand $E(D_i)$	Expected Leadtime in Days $E(L_i)$	Variance Leadtime in Days $V(L_i)$	Expected Leadtime Demand $E(X)$	Variance & St. Dev. Leadtime Demand $V(X); \sigma(X)$
23 (Pad Assy-1) FSN: 1095009120256	56.6 (37.4)*	384.6	73984	21,752.	236,712,811; 15,385.5
37 (20 mm. Gun Ba) FSN: 1005007879802	39.9 (24.2)*	459.3	34615	18,323	55,109,982; 7423.6
39 (Guide Rolle) FSN: 1005007545293	6.3 (3.2)*	145.3	3698	915.5	147,747; 384.4
50 (Lead tape) FSN: 7045008479020	166.7 (2.9)*	149.6	3977	24,982	110,876,427; 10,530
53 (Adapter) FSN: 1560009492087JH	7.9 (4.1)*	312.1	28690	2,478	1,810,423; 1345.5

( )* represents 1980 demand data

TABLE 5.3

GOODNESS OF FIT OF LEADTIME DEMAND TO THE NORMAL AND THE LAPLACE--1977-78 DATA

Item No: FSN	Sample Size	Entire Distribution: Normal D_{max} [1]	Entire Distribution: Laplace D_{max} [1]	Right Half: Normal D_{max} [1]	Right Half: Laplace D_{max} [1]
23: 1095009120256	30	0.1568	0.1143	0.0938	0.1143
(Leadtime normal)	100	0.0998	0.1403*	0.0800	0.1403**
37: 1005007879802	30	0.0784	0.1326	0.0784	0.1001
(Leadtime gamma)	100	0.0743	0.1363*	0.0742	0.0688
39: 1005007545293	30	0.1900	0.1941	0.1190	0.0759
(Leadtime lognormal)	100	0.0858	0.1117	0.0858	0.0755
50: 7045008479020	30	0.2171	0.2791*	0.1683	0.1181
(Leadtime gamma)	100	0.1416*	0.1851	0.0901	0.1015
53: 1560009492087	30	0.0955	0.1450	0.0531	0.0715
(Leadtime gamma)	100	0.1745*	0.2283*	0.1311	0.0955

[1] From the Table of the Kolmogorov-Smirnov Statistic, two-sided test, the 0.95 critical value for n=30 is 0.242, that for n=100 is 0.136 (Conover, 1971, p. 397).

*Reject the hypothesis of good fit at the ninety-five percent significance level.

**at k = 0.37; beyond that, the fit is quite good.

TABLE 5.4

MEANS AND VARIANCES OF LEADTIME DEMAND:
1980 DATA

Item no.	Expected Daily Demand $E(D_i)$	Expected Leadtime in Days $E(L_i)$	Expected Leadtime Demand $E(X)$	Variance and Standard Deviation of Leadtime Demand $V(X)$; $\sigma(X)$
23	37.4	384.6	14,384.04	103,500,305.7; 10,173.51
37	24.2	459.3	11,115.06	20,283,043.47; 4503.67
39	3.2	145.3	464.96	27,486.32; 195.79
50	2.9	149.6	433.84	33,881.76; 184.07
53	4.1	312.1	1,279.61	434,781.98; 659.38

TABLE 5.5

FIT OF LEADTIME DEMAND TO THE NORMAL AND THE LAPLACE

Item No.	Sample Size	k	Entire Distribution: Normal D_{max}[1]	Entire Distribution: Laplace D_{max}[1]	k	Right Tail: Normal D_{max}[1]	Right Tail: Laplace D_{max}[1]
23	30	$-\infty$ to $+\infty$	0.1594	0.1174	0 to $+\infty$	0.0986	0.1174
					1.00 to $+\infty$	0.0986	0.1174
					1.64 to $+\infty$	0.0173	0.0241
37	30	$-\infty$ to $+\infty$	0.1596	0.1877	0 to $+\infty$	0.0731	0.1163
					1.00 to $+\infty$	0.0608	0.0799
					1.64 to $+\infty$	0.0324	0.0271
39	30	$-\infty$ to $+\infty$	0.1686	0.1893	0 to $+\infty$	0.1471	0.1143
					1.00 to $+\infty$	0.0349	0.0562
					1.64 to $+\infty$	0.0329	0.0290
50	30	$-\infty$ to $+\infty$	0.2089	0.2547*	0 to $+\infty$	0.1876	0.1661
					1.00 to $+\infty$	0.0646	0.0731
					1.64 to $+\infty$	0.0646	0.0580
53	30	$-\infty$ to $+\infty$	0.1510	0.1946	0 to $+\infty$	0.1510	0.1946
					1.00 to $+\infty$	0.1155	0.1504
					1.64 to $+\infty$	0.0938	0.0916

[1] From the Table of the Kolmogorov-Smirnov Statistic, two-sided test, the 0.95 significance level for n=30 is 0.242 (Conover, 1971, p. 397).

*Reject the hypothesis of good fit at the ninety-five percent significance level.

6. COST-BENEFIT ANALYSIS

In doing the cost-benefit analysis, we compare the current system at AFLC with an optimal system. Because of arguments presented in the previous section, we choose the Presutti-Trepp Model IV (1970, pp. 248-249) as the optimal system. One simulation, using 1980 data and with $\lambda = 660$, $Z_i = 0.5$, $a_i = 0.23$, is presented in Table 6.1, which is sorted by FSN for convenience. And based on the same parameters and upon an understanding of the present procedures at LORRA, Table 6.2 presents a description of the current system. How each column in these tables is calculated may be respectively seen in Appendices B and C of this report.

The question concerning the mix of items at an ALC and how that mix would change is answered by comparing the corresponding columns in Tables 6.1 and 6.2. The mix will change.

It may be worthwhile to compare the current system with the equivalent optimal system. This comparison is presented in Table 6.3 in terms of several attributes: service level, number of backorders, fill ratio, value of safety stock, and so on.

It may also be worthwhile to study the behavior of the optimal system for different shortage factors, λ_i, and different item essentiality, Z_i. Table 6.4 presents this behavior for $Z_i = 0.5$ and for $\lambda = 660, 600, 500, 400, 300, 200$. Table 6.5 gives a similar comparison for $Z_i = 1$

From Tables 6.4 and 6.5 we produce the exchange curves in Figures 6.1 and 6.2. These exchange curves should give an idea of the additional investment in safety stock required to reduce the number of backorders or to improve the service levels. The reader should keep in mind that these tables and exchange curves are based on a sample of size $n = 56$. This sample represents

TABLE 6.1. OPTIMAL SYSTEM

(1) Item #	(2) FSN	(3) Daily Demand	(4) Mean Leadtime Days	(5) Variance Leadtime	(6) Mean Leadtime Demand	(7) Variance Leadtime Demand	(8) Std. Dev. Leadtime Demand $\sigma^*(X)$	(9) Unit Cost c_i	(10) The Wilson EOQ Q_w	(11) Optimal EOQ Q^*	(12) Optimal Safety Factor, k_i^*
48	1005000178809	21.70	183.25	535.00	3976	255970	505.84	3.08	3118.9	3497.0	2.242
46	1005000511579	15.51	215.50	5305.67	3342	1279399	1131.11	7.08	1739.1	2718.0	2.378
47	1005003357318	4.92	411.40	51911.07	2025	1259797	1122.41	27.08	501.0	1732.2	1.681
45	1005006999882	13.53	300.88	48835.21	4072	8876782	2979.39	9.64	1392.3	4632.0	2.407
43	1005006999923	12.86	251.70	18379.22	3237	3043259	1744.49	10.64	1291.9	3019.8	2.281
44	1005006999931	15.94	276.33	4629.44	4404	1180079	1086.31	2.93	2229.0	3125.8	2.885
42	1005007010793	14.23	289.17	6582.08	4116	1337814	1156.64	8.72	1591.3	2527.5	2.288
41	1005007545266	6.09	176.39	4397.36	1077	349340	591.05	2.39	1528.6	2002.6	2.923
40	1005007545267	1.79	367.50	27855.61	657	89677	299.46	41.36	244.3	535.0	1.303
39	1005007545293	3.24	145.29	3697.86	471	39265	198.15	5.66	721.0	876.6	2.126
38	1005007755579	3.03	293.00	16962.46	888	156687	395.84	7.26	617.5	957.9	2.354
37	1005007979802	24.21	459.29	34614.60	11121	20306279	4506.25	220.55	389.3	6396.5	.239
36	1005007889718	2.54	188.67	1051.70	479	7259	85.20	9.12	504.3	568.1	1.500
35	1005009918230	.48	203.00	10310.37	98	2520	50.20	182.94	60.5	105.6	.158
34	1095001111640	.78	235.33	24339.12	183	14902	122.08	65.84	103.9	221.4	.966
33	1095004335657	4.26	201.50	875.57	859	16756	129.44	111.22	230.0	339.1	.375
32	1095004913487	33.62	253.00	8508.22	8505	9624237	3102.30	3.27	3767.6	6553.4	3.001
31	1095005168069	15.75	208.43	5468.60	3282	1359378	1165.92	1.43	3171.7	4101.6	3.257
30	1095005227703	14.93	169.17	5548.76	2525	1238763	1113.00	2.21	2483.9	3392.6	3.046
29	1095006614744	1.62	290.00	2084.84	469	5927	76.99	46.12	220.1	281.2	.776
28	1095003747369	2.98	272.50	35626.56	812	317048	561.07	63.45	254.6	870.8	1.077
27	1095009120243	10.68	260.00	3255.84	2776	373890	611.47	11.79	1118.2	1631.2	1.950
23	1095009129256	37.39	384.57	73984.00	14380	103454128	10171.24	9.47	2334.9	14753.8	2.455
26	1095009553210	2.54	408.17	146773.27	1039	951195	975.29	30.93	337.0	1457.2	1.603
25	1095010133419	52.45	147.29	2466.12	7725	6791670	2606.08	3.61	4478.9	6685.9	2.812
24	1270000831981	1.07	248.60	3254.70	267	4028	63.47	201.97	85.7	141.6	.053
61	1443005727651	2.39	289.56	9868.44	692	57077	238.91	23.20	377.2	582.2	1.529
60	1560003101191	5.25	217.29	4399.67	1140	122345	349.78	5.53	931.2	1210.8	2.312
59	1560004390187	.84	270.13	5521.98	226	4085	63.91	77.04	122.4	175.7	.603
58	1560006099968	18.81	179.17	1058.20	3371	377949	614.77	1.43	3466.9	3928.7	2.840
57	1560006117049	2.28	307.17	7440.79	700	39358	198.39	32.85	309.5	480.1	1.287
56	1560006220532	1.94	209.86	7637.01	408	29239	171.00	8.07	469.0	605.2	2.030
55	1560006284732	4.02	139.57	14570.90	562	236620	486.44	13.07	530.4	976.1	2.052
54	1560007303854	.98	254.50	1823.29	250	2003	44.76	108.31	111.8	147.8	.240
53	1560009892087	4.05	312.14	28689.58	1263	470915	686.23	30.87	425.4	1130.5	1.554
52	1670007970253	1.36	276.80	8828.49	376	16665	129.09	9.52	361.0	463.7	1.903
62	2530008269468	2.21	197.50	13280.26	436	65200	255.34	6.44	559.7	768.7	2.298
51	3040006211345	2.22	258.33	4900.00	572	24620	156.91	25.44	346.8	475.0	1.323
21	3110006999957	1.47	350.25	5004.15	516	11384	106.69	535.94	61.6	172.9	.000
20	3110003429451	4.95	332.72	3596.40	1647	89768	299.61	48.05	377.1	644.4	1.090
19	4210002727815	1.83	183.33	4361.28	336	14995	122.45	207.16	110.5	227.0	.143
18	4310010339204	1.03	463.25	9156.58	479	10230	101.14	201.65	84.1	181.9	.179
17	4440005749186	1.41	465.72	51524.46	657	103238	321.31	65.60	172.3	512.4	1.038
22	5365007855943	3.63	230.00	2609.17	835	35228	187.69	18.75	517.1	666.5	1.431
12	5821002694508	.77	154.13	2653.28	118	1678	40.96	142.20	86.3	120.0	.128
13	5821009906461	12.46	210.18	2152.03	2620	336941	580.47	2.74	2038.5	2489.9	2.661
1	5826000146411	1.06	379.00	4046.23	401	4933	70.24	65.10	149.8	207.5	.675
11	5831008803563	5.62	275.55	71497.41	1548	2258144	1502.71	71.38	329.7	2175.1	1.027
2	5941000738392	2.61	234.17	6041.95	612	41854	204.58	31.65	337.6	512.0	1.292
10	5895000894403	.98	284.67	11718.06	278	11448	106.99	338.76	63.1	174.2	.000
15	5995001167508	4.30	261.80	9211.37	1125	171575	414.22	283.25	144.7	619.6	.036
9	5895008451349	8.29	234.33	9510.15	1942	655151	809.41	3.72	1426.6	2109.5	2.780
8	5895009497160	.57	248.50	10805.60	141	3611	60.09	160.91	69.7	124.1	.260
7	5960004009106	4.55	270.10	62490.00	1230	1296256	1138.53	91.38	262.3	1651.8	.852
3	5962004537749	7.90	338.72	131965.09	2676	8241398	2870.78	1.07	2597.3	5326.4	3.866
50	7045008479020	2.91	149.63	3976.56	415	34102	184.67	25.39	397.8	549.2	1.336
	TOTALS	430.83	14818.50	1062211.62	110377	177103950	49070.76	3726.22	51665.0	102529.4	86.902
	AVERAGES	7.69	264.62	18968.06	1971	3162571	876.26	66.54	922.6	1830.9	1.552

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TABLE 6.1. OPTIMAL SYSTEM (Continued)

(13) Safety Stock, $K^{*}\sigma^{*}$	(14) \$s Safety Stock	(15) Service Level	(16) Weighted Backorders B_T	(17) Fill Ratio	(18) \$s Inventory on Hand	(19) Ordering Cost Annual	(20) Annual Holding Cost Inventory Position	(21) Annual Operating Cost	(22) Annual no. of Backorders B_N	(23) FSN	(24) Item #
1134.07	3492.92	.979	.38	.998	8880.66	945.25	4858.80	5344.06	16.77	1005000178809	48
2689.26	19939.49	.983	1.97	.995	28675.51	907.33	12031.13	12338.46	27.55	1005000511579	46
1886.29	51080.82	.954	7.49	.981	74940.61	451.21	29755.85	30207.06	13.45	1005003357318	47
7171.90	69137.12	.983	7.08	.993	91599.88	463.97	30065.72	30529.68	32.74	1005006999882	45
3978.52	42331.51	.980	4.57	.993	58493.99	676.26	21353.10	22029.35	34.33	1005006999923	43
3153.90	9182.44	.992	.78	.998	13766.35	535.59	6132.79	6668.38	11.72	1005006999931	44
2615.93	23372.55	.980	2.49	.994	34135.80	894.26	16096.79	16991.06	31.14	1005007016793	42
1727.45	4111.33	.992	.35	.998	6496.13	319.35	2183.20	2502.54	3.64	1005007545266	41
330.27	16111.70	.921	3.05	.971	27458.81	530.53	12507.06	13037.59	18.55	1005007545267	40
421.29	2384.49	.975	.28	.996	4868.40	388.17	1731.63	2119.81	4.60	1005007545293	39
931.99	6766.26	.982	.71	.995	10253.74	332.38	3838.75	4171.13	5.52	1005007755579	38
1078.33	237302.58	.644	244.90	.846	1051202.29	601.08	781072.54	781673.62	1339.95	1005007879802	37
127.79	1165.42	.940	.19	.994	3759.61	469.49	1868.73	2338.22	5.81	1005007883718	36
7.94	1452.51	.600	2.26	.872	11942.36	728.57	6696.06	7424.63	22.25	1005008938230	35
117.96	7766.34	.873	1.98	.954	15314.99	369.04	6233.80	6602.83	12.85	1095001111640	34
46.49	5491.98	.706	3.55	.922	25036.63	1995.41	27540.21	29535.62	118.91	1095004335657	33
9311.38	30848.20	.993	2.50	.998	41175.32	814.55	15964.36	16778.91	27.58	1095004918487	32
3797.92	5830.89	.995	.41	.999	8364.67	403.34	3003.13	3406.47	5.65	1095005168069	31
1390.43	7192.84	.993	.61	.998	11244.38	462.29	3869.09	4331.30	8.28	1095005227703	30
59.71	2753.73	.933	.87	.968	9318.02	913.76	7101.88	8015.64	18.72	1095008614784	29
636.69	33494.47	.991	8.80	.956	67236.47	543.29	27055.44	27598.73	47.43	1095008747369	28
1132.18	18055.82	.968	1.78	.992	23713.81	1039.24	12971.81	14011.05	31.58	1095009120243	27
24966.66	236334.30	.984	23.74	.993	306743.18	402.42	101767.99	102170.41	88.85	1095009120256	23
1563.37	49355.04	.948	7.43	.978	71350.70	377.24	23692.83	24070.07	19.75	1095009553210	26
7327.37	26451.79	.991	2.32	.997	38536.65	1245.61	15273.78	16519.39	47.51	1095010133419	25
3.35	676.67	.536	3.16	.859	16256.61	1205.09	15859.91	17064.99	54.48	1270000831981	24
365.20	3472.53	.942	1.37	.984	15289.56	651.91	7195.31	7847.22	13.91	1440005727651	61
808.68	4472.00	.981	.48	.996	7825.06	455.41	3249.13	3704.54	7.28	1560003101191	60
38.55	2969.66	.787	1.21	.946	9923.70	755.62	6240.95	6996.57	16.16	1560004390187	59
1745.97	2991.74	.991	.22	.999	5106.36	503.10	2329.01	2832.11	6.75	1560006099968	58
255.33	9387.60	.919	1.63	.977	16379.30	753.84	9032.81	9786.65	18.79	1560006117049	57
347.04	2430.62	.972	.34	.994	5248.22	337.26	1962.67	2299.93	3.93	1560006220512	56
938.14	13945.75	.973	1.57	.991	19465.71	433.19	6156.43	6589.62	13.20	1560006284732	55
10.75	1164.27	.644	1.19	.925	9429.43	1053.20	8326.70	9379.90	26.65	1560007803854	54
1066.18	32419.30	.944	5.22	.978	50691.63	569.26	20551.80	21120.05	31.34	1560009492087	53
245.62	2318.31	.966	.30	.993	4551.17	307.76	1868.70	2176.46	3.24	1670007970253	52
536.85	3779.29	.981	.41	.996	6259.56	301.82	2084.52	2386.34	3.57	2530008269468	62
207.60	5281.35	.923	.98	.982	11373.64	740.52	5952.99	6693.51	14.14	3040006211345	51
.00	.00	.500	7.40	.804	54250.67	1353.73	74277.51	75631.24	104.07	3110006994957	21
326.68	15697.17	.893	3.55	.967	31520.69	1219.65	25372.77	26592.42	59.68	3110009429451	20
17.57	3638.83	.592	6.25	.856	29741.90	1282.38	22359.27	23641.65	95.29	4210002727815	19
18.15	3660.08	.612	5.03	.859	24025.90	900.92	27232.60	28133.52	52.21	4310010339204	18
333.54	21490.35	.885	5.19	.954	39367.23	437.28	18812.53	19249.81	23.22	4440005749186	17
268.61	5036.50	.934	.87	.987	11317.92	864.90	6196.81	7061.71	17.08	5365007855943	22
5.25	745.92	.583	1.44	.901	9684.28	1014.68	5998.25	7012.93	27.35	5821002694508	12
1544.37	4231.57	.988	.19	.998	7644.85	525.89	3408.75	3934.64	8.57	5821009906461	13
47.40	3085.72	.807	1.13	.955	9986.68	809.85	8268.99	9078.84	17.29	5826000146411	1
1543.89	110203.07	.883	26.43	.950	191606.36	410.12	68616.38	69026.50	100.62	5831008803563	11
264.38	8367.65	.920	1.60	.979	16570.37	810.33	8241.61	9051.94	20.75	5841000738392	2
.00	.00	.500	7.39	.805	34509.41	890.13	28439.99	29330.12	68.70	5895000894403	10
15.10	4276.05	.525	28.91	.803	108404.53	1101.20	94454.04	95555.24	305.39	5895001167508	15
2250.47	8373.75	.990	.74	.997	12300.90	412.74	4489.55	4902.29	7.74	5895004451349	9
15.60	2510.70	.654	2.38	.888	13265.57	724.78	8086.25	8811.03	22.88	5895004497160	8
969.59	88600.77	.850	25.64	.936	168755.60	437.62	63579.83	64016.45	104.36	5960004009106	7
11099.80	11876.79	.998	.76	.999	14728.03	155.84	4045.72	4201.56	2.12	5962004537739	3
246.75	6265.04	.924	1.16	.982	13296.38	841.18	5587.12	6428.30	18.54	7045008479020	50
105353.53	1297888.92	48.677	474.79	53.621	3013890.09	39009.75	1742644.37	1781654.12	3284.44	TOTALS	
1881.31	23169.45	.869	8.48	.958	53812.32	696.60	31118.65	31815.25	58.65	AVERAGES	

TABLE 6.2. CURRENT SYSTEM

(1) Item no.	(2) FSN	(3) Monthly Demand	(4) Months Leadtime	(5) Leadtime Demand	(6) Quarterly MAD	(7) Std. Dev. Leadtime Demand σ	(8) Q	(9) Unit Cost	(10) K	(11) Safety Stock Kσ
48	1005000178809	650.96	6.11	3976	483.60	985.39	3906	3.08	2.63	2594.27
46	1005000511579	465.25	7.18	3342	923.06	2132.28	2791	7.08	2.71	3342.05
47	1005003357318	147.67	13.71	2025	128.75	510.46	886	27.08	1.62	826.23
45	1005006999882	406.04	10.03	4072	371.94	1127.43	2436	9.64	2.22	2506.74
43	1005006999923	385.83	8.39	3237	215.00	562.40	2315	10.64	1.73	972.91
44	1005006999931	478.08	9.21	4404	201.25	568.30	2868	2.93	2.50	1420.27
42	1005007016793	427.04	9.64	4116	139.88	322.20	2562	8.72	1.41	453.37
41	1005007545266	182.63	5.90	1077	124.40	246.80	1490	2.38	2.52	621.89
40	1005007545267	53.63	12.25	657	73.07	262.61	322	41.36	1.49	391.63
39	1005007545293	97.17	4.84	471	85.18	146.59	723	5.66	2.05	300.51
38	1005007755579	90.92	9.77	888	102.25	303.14	633	7.26	2.44	741.02
37	1005007879802	726.42	15.31	11121	666.50	2912.12	4359	220.55	.21	620.52
36	1005007889718	76.17	6.29	479	32.26	171.38	533	9.12	2.07	354.64
35	1005034334230	14.54	6.77	98	19.88	43.82	87	162.94	.19	8.36
34	1095001111640	23.33	7.84	183	14.75	36.54	140	65.84	.49	17.94
33	1095034335657	127.83	6.72	859	68.25	149.59	767	111.22	.00	.00
32	1095000918487	1008.54	8.43	8505	1042.13	2842.51	6051	3.27	3.00	8505.35
31	1095005167069	472.42	6.95	3282	298.32	671.31	3118	1.43	3.00	2013.93
30	1095005227703	447.79	5.64	2525	344.53	661.04	2637	2.21	2.85	1884.00
29	1095008614744	48.54	9.67	469	39.72	116.75	291	46.12	1.03	120.02
28	1095008747369	89.38	9.08	812	116.66	381.49	536	63.45	1.12	429.15
27	1095009120243	320.29	8.67	2776	151.88	945.12	1922	11.79	2.12	2000.94
23	1095009120256	1121.75	12.82	14380	671.06	2508.51	6730	9.47	2.10	5270.27
26	1095009553210	76.33	13.61	1039	66.00	259.87	458	10.93	1.52	394.04
25	1095010133419	1573.46	4.91	7725	525.47	911.09	9441	1.61	1.84	1679.65
24	1270000331981	32.25	8.29	267	19.25	49.45	193	201.97	.00	.00
61	1440005727651	71.71	9.65	692	37.16	109.09	430	23.20	1.21	131.81
60	1560003101191	157.46	7.24	1140	43.47	194.09	945	5.53	2.08	402.81
59	1560004390187	25.09	9.00	226	24.50	67.90	150	77.04	.74	50.15
58	1560006099968	564.42	5.97	3371	320.75	642.51	3522	1.43	2.95	1894.31
57	1560006117049	68.38	10.24	700	37.09	114.40	410	32.85	1.03	117.59
56	1560006229532	58.29	7.00	408	45.88	103.80	476	8.07	1.85	192.09
55	1560006294732	120.75	4.65	562	74.75	124.71	724	13.07	1.34	167.50
54	1560007603854	29.42	8.48	250	28.07	74.09	177	108.31	.45	33.57
53	1560009492087	121.38	10.40	1263	53.98	168.45	728	30.97	.94	158.77
52	1670007970253	40.75	9.23	376	116.13	104.95	372	9.52	2.63	375.99
62	2530008269468	66.25	6.58	436	70.19	151.47	541	6.44	2.18	330.71
51	3040006211345	66.46	8.61	572	30.03	80.23	399	25.44	.98	78.77
21	3110006999957	44.21	11.67	516	22.38	77.17	265	535.94	.00	.00
20	3110009329451	148.50	11.09	1647	172.88	570.53	891	48.05	1.27	724.01
19	4210002727815	55.00	6.11	336	88.50	190.39	330	207.16	.15	27.21
18	4310011139204	30.96	15.44	478	26.13	115.04	186	201.65	.24	27.32
17	4440005749186	42.33	15.52	657	32.25	142.66	254	65.60	.98	139.67
22	5365007855943	108.92	7.67	835	70.25	170.88	654	18.75	1.38	235.86
12	5821002694508	23.00	5.14	118	25.50	45.69	138	142.20	.11	4.92
13	5921009906461	373.92	7.01	2620	540.75	1224.84	2244	2.74	3.00	2619.68
1	5826000146411	31.75	12.63	401	14.56	53.74	190	65.10	.55	29.68
11	5831008301563	168.54	9.19	1548	68.13	191.94	1011	71.38	.21	40.54
2	5841000738392	78.39	7.81	612	63.88	157.64	470	31.65	1.18	185.75
10	5895000894403	29.29	9.49	278	29.09	84.19	176	338.76	.00	.00
15	5895001167508	128.91	8.73	1125	118.13	319.08	773	243.25	.00	.00
9	5895004451349	248.63	7.81	1942	240.40	593.56	1492	3.72	2.80	1664.05
8	5895009497160	17.00	8.28	141	17.75	45.95	102	160.91	.22	9.96
7	5960004009106	136.57	9.00	1230	114.75	318.00	819	91.38	.52	166.61
3	5962004537739	237.04	11.29	2676	219.85	736.68	2532	1.07	3.00	2210.05
50	7945000479020	87.29	4.99	435	115.06	201.77	524	25.39	1.42	287.50
	TOTALS	12924.85	493.95	110377	10127.38	27274.03	80141	3726.22	80.28	49776.59
	AVERAGES	230.80	8.82	1971	180.85	487.04	1431	66.54	1.43	888.87

TABLE 6.2. CURRENT SYSTEM (Continued)

(12) $ Value Safety Stock	(13) True K	(14) Actual Weighted B_1	(15) Actual Service Level	(16) $ Inventory on Hand	(17) Fill Ratio	(18) Annual Ordering Cost	(19) Annual Holding Cost (Inventory Position)	(20) Annual Operating Cost	(21) Annual Units Backordered	(22) FSN	(23) Item no.
7593.34	5.129	.01	1.00	14005.25	1.000	882.14	6034.00	6920.14	.25	1005000178809	48
23661.68	5.107	.04	1.00	33544.17	1.000	882.14	13157.21	14039.35	.57	1005000511579	46
22174.21	.736	42.21	.82	36656.76	.894	882.14	20519.12	21403.26	188.47	1005003357318	47
24165.00	.341	94.98	.85	37738.87	.910	882.14	17287.89	18170.03	439.33	1005006999882	45
10351.79	.558	63.24	.77	24013.25	.897	882.14	13135.38	14017.52	474.74	1005006999923	43
4161.40	1.397	7.90	.92	8410.02	.979	583.64	4891.24	5474.88	118.02	1005006999931	44
3951.40	.392	35.86	.71	15750.13	.912	882.14	11734.22	12616.36	449.35	1005007016793	42
1180.09	1.352	6.43	.89	3284.11	.969	429.15	1337.77	1766.92	67.43	1005007545266	41
10197.67	1.308	4.28	.92	23206.23	.960	882.14	11505.58	12387.72	26.02	1005007545267	40
1700.47	1.517	.79	.94	3755.75	.989	470.68	1474.38	1945.06	13.16	1005007545293	39
5373.77	1.872	1.96	.96	7705.36	.996	503.14	3248.43	3751.57	15.31	1005007755579	38
130855.41	.138	357.26	.59	775078.90	.776	882.14	706164.98	707047.12	1954.72	1005007879802	37
3234.36	4.163	.01	1.00	5526.86	1.000	530.58	2275.98	2806.56	.15	1005007889718	36
1528.65	.166	2.61	.60	10463.22	.853	882.14	6326.72	7209.86	25.65	1005008938230	35
1181.34	.147	9.67	.59	6931.71	.799	583.64	4102.91	4686.55	56.27	1095001111640	34
.00	.000	2.73	.50	43259.05	.940	882.14	31773.19	32655.33	91.51	1095004335657	33
27812.51	2.746	3.83	.99	37731.35	.997	882.14	15069.32	15951.46	42.29	1095004918487	32
2875.91	1.727	4.63	.96	5122.50	.989	530.58	2254.65	2785.23	63.66	1095005168069	31
4163.65	1.693	5.09	.95	7154.98	.987	583.64	2923.98	3507.62	69.47	1095005227703	30
3535.48	1.559	.28	.94	12277.23	.990	882.14	7795.14	8677.28	5.97	1095008614744	29
27223.56	.762	19.61	.93	46604.64	.907	882.14	22023.90	22906.04	100.27	1095008747369	28
23591.04	3.272	.23	1.00	34925.23	.995	882.14	15558.79	16440.93	4.18	1095009120243	27
49905.46	.518	561.15	.76	92406.59	.844	882.14	50129.49	51011.62	2100.53	1095009120256	23
12167.63	.304	71.15	.72	23671.54	.794	882.14	11820.11	12702.25	188.99	1095009553210	26
6063.53	.645	35.93	.80	23363.51	.961	882.14	11728.15	12610.29	736.26	1095010133419	25
.00	.000	2.57	.50	20577.70	.886	882.14	16908.70	17790.84	44.28	1270000831981	24
3058.09	.552	7.00	.77	8374.10	.917	882.14	5544.58	6426.72	71.36	1440005727651	61
2227.52	1.152	3.11	.90	4874.14	.975	583.64	2563.73	3147.37	47.46	1560003101191	60
3663.28	.745	1.08	.84	9625.99	.952	882.14	6223.25	7105.39	14.37	1560004390187	59
2708.87	3.181	.17	.99	5227.58	.999	561.19	2310.92	2872.11	5.35	1560006099968	58
3862.78	.593	4.91	.78	10524.05	.930	882.14	7729.30	8610.44	57.41	1560006117049	57
1559.14	1.123	1.54	.90	3494.21	.975	429.15	1554.80	1983.94	17.80	1560006220532	56
2189.24	.344	22.03	.69	7499.80	.872	583.64	3281.22	3864.86	185.64	1560006284732	55
3036.46	.750	.49	.83	13301.85	.969	882.14	9252.41	10134.55	10.91	1560007803854	54
4901.24	.233	45.28	.64	18937.85	.813	882.14	12679.56	13561.70	271.84	1560009492087	53
3573.39	7.336	.00	1.00	5348.40	1.000	383.97	2053.39	2437.37	.00	1670007970253	52
2129.79	1.295	2.29	.92	3909.06	.975	429.15	1536.24	1965.39	20.20	2530008269468	62
2301.43	.532	3.69	.75	7263.92	.933	882.14	4976.08	5858.22	53.05	3040006211345	51
.00	.000	5.20	.50	76660.80	.862	882.14	79972.86	80855.00	73.20	3110006999957	21
34789.77	2.416	.41	.98	56734.14	.996	882.14	31126.28	32008.42	6.94	3110009429451	20
5037.74	.222	4.06	.63	41499.76	.906	882.14	25172.74	26054.88	61.84	4210002727815	19
5209.47	.270	4.35	.66	25992.38	.878	882.14	27747.74	28629.88	45.18	4310010339204	18
9162.62	.435	18.49	.73	19919.37	.837	882.14	13938.22	14820.36	82.69	4440005749186	17
4422.40	1.257	1.13	.92	10591.57	.983	882.14	6027.47	6909.61	22.28	5365007855943	22
693.68	.120	1.27	.58	10873.04	.912	882.14	6282.39	7164.53	24.23	5821002694508	12
7177.93	6.330	.00	1.00	10251.57	1.000	583.64	4008.78	4592.42	.05	5821009906461	13
1932.09	.923	1.74	.72	8359.72	.930	882.14	7876.35	8758.49	26.73	5826000146411	1
2393.70	.027	164.95	.52	62532.62	.690	882.14	34381.31	35263.45	627.91	5831008803563	11
5873.14	.908	2.96	.86	13508.77	.959	882.14	7517.56	8399.70	38.51	5841000712392	2
.00	.900	7.34	.50	34743.04	.806	882.14	28501.45	29383.59	68.24	5895000894403	10
.00	.000	25.75	.50	124129.32	.824	882.14	98482.47	99364.61	272.01	5895001167508	15
6190.26	2.956	2.78	.97	8585.63	.990	583.64	3723.56	4307.20	28.95	5895004451349	9
1602.59	.166	3.18	.60	10833.26	.850	882.14	7467.58	8349.72	30.56	5895009497160	8
15225.19	.146	102.67	.59	71429.08	.745	882.14	37955.52	38837.66	418.02	5960004009106	7
2364.75	.770	97.63	.83	3528.07	.904	327.89	1514.05	1841.94	273.61	5962004537739	3
7299.52	1.557	.88	.94	13993.28	.986	882.14	5750.58	6632.72	14.18	7045008479020	50
570085.49	73.106	1868.84	44.59	2056602.30	51.586	41088.41	1490335.65	1541424.07	10147.26	TOTALS	
10180.10	1.305	33.37	.80	36725.04	.921	769.36	26755.99	27525.36	181.20	AVERAGES	

TABLE 6.3

CURRENT VERSUS OPTIMAL SYSTEM
$\lambda=660$, $\sqrt{R}=2$, $Z_i=0.5$, $n=56$, WR-ALC

	Current	Optimal	Difference
1. Average Service Level	79.6%	86.9%	7.3%
2. No. of Units backordered at any time, β_T	3,738	950	-2788
3. Annual number of backorders, β_N	10,147	3284	-6863
4. Fill ratio	92.1%	95.8%	3.7%
5. Value of Average Inventory on Hand	$1,923,768	$3,013,490	$1,089,722
6. Value of Safety Stocks	$570,085	$1,297,489	$727,404
7. Annual Operating Cost	$1,541,420	$1,781,654	$240,234
8. Two Months of Demand	25,850	25,850	--
9. Safety Stock, units	49,777	105,354	55,577

TABLE 6.4

OPTIMAL SYSTEM, $Z_i = 1/2$

	$\lambda=660$	$\lambda=600$	$\lambda=500$	$\lambda=400$	$\lambda=300$	$\lambda=200$
1. Service Level	.869	.859	.840	.822	.798	.757
2. No. of Units Backordered at any time, β_T	950	1039	1225	1375	1552	1902
3. Annual no. of Backorders, β_N	3284	3580	4170	4670	5251	6369
4. Fill Ratio	.958	.954	.948	.943	.936	.924
5. Value of Inventory on Hand	$3,013,490	$2,905,441	$2,712,119	$2,577,232	$2,451,861	$2,281,986
6. Value of Safety Stocks	$1,297,488	$1,175,499	$954,049	$805,215	$672,074	$487,044
7. Annual Operating Cost	$1,781,654	$1,753,596	$1,702,663	$1,668,431	$1,637,809	$1,595,252
8. Two Months of Demand in Units	25,850	25,850	25,850	25,850	25,850	25,850
9. Safety Stock in Units	105,354	102,075	95,850	88,795	79,979	67,590

TABLE 6.5

OPTIMAL SYSTEM, $Z_i=1$

	$\lambda=660$	$\lambda=600$	$\lambda-500$	$\lambda=400$	$\lambda=300$	$\lambda=200$
1. Service Level	.930	.923	.909	.889	.859	.822
2. No. of Units Backordered at any time, β_T	483	531	634	789	1039	1375
3. Annual no. of Backorders, β_N	1696	1861	2213	2740	3580	4670
4. Fill Ratio	.977	.975	.970	.964	.954	.943
5. Value of Inventory on Hand	$3,867,276	$3,745,094	$3,516,640	$3,241,695	$2,905,441	$2,577,232
6. Value of Safety Stocks	$2,227,341	$2,097,106	$1,851,772	$1,551,507	$1,175,499	$805,215
7. Annual Operating Cost	$1,995,520	$1,965,566	$1,909,139	$1,840,078	$1,753,596	$1,668,431
8. Two Months of Demand in Units	25,850	25,850	25,850	25,850	25,850	25,850
9. Safety Stock in Units	129,342	126,038	119,726	112,000	102,075	88,795

Figure 6.1. Exchange Curves for Safety Stock versus Expected Number of Units Backordered at any Time, β_T

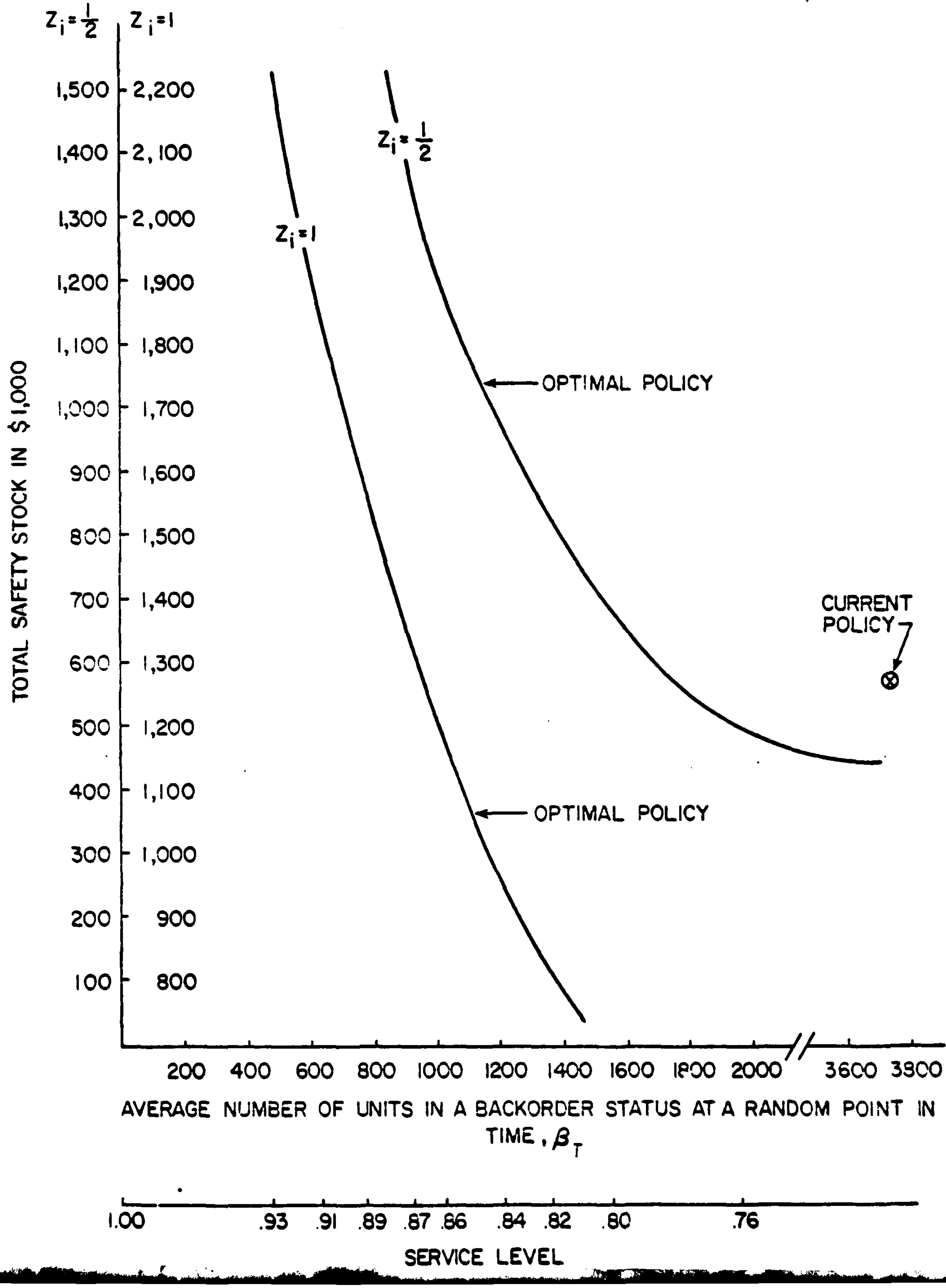
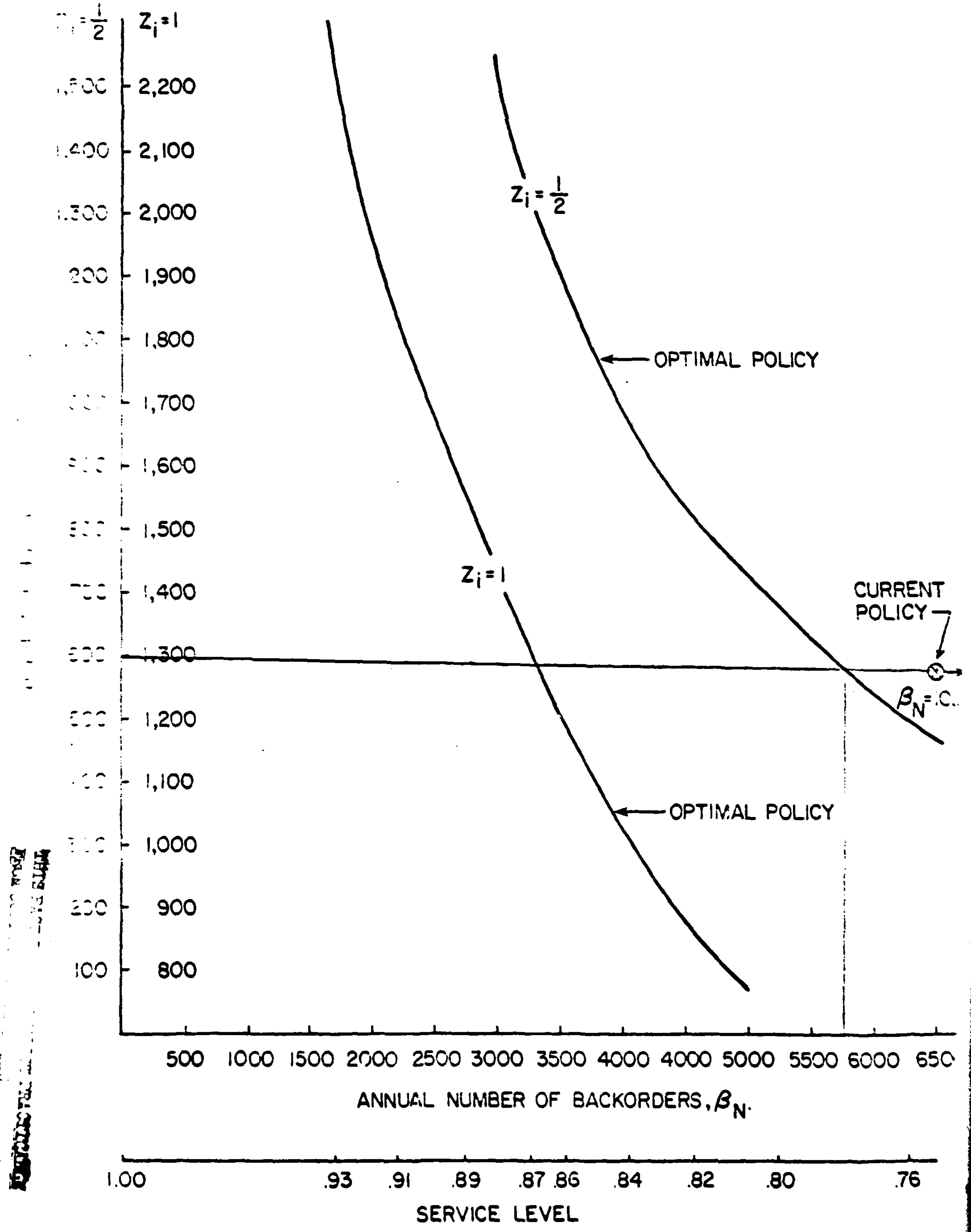


Figure 6.2. Exchange Curves for Safety Stock Versus Annual Number of Backorders, β_N



an annual procurement of $4.9 million. Since total annual procurement is one billion dollars, our sample represents about one-half of one percent of total procurement. Consequently, the numbers in this section must be multiplied by 200 in order to fathom the impact on the total inventory system. Returning to Table 6.3, for example, we see that to bring the system to optimality would require a one-time additional procurement for safety stocks of $727,400 x 200 ≈ $150 million. We can also see that the annual operating cost will increase by $240,234 x 200 ≈ $50 million. This is the price to pay in order to reduce backorders by 6863 x 200 ≈ 1,400,000 units per year.

In Figures 6.1 and 6.2, we see that the current policy itself is not on the optimal curve. Thus, AFLC can presently reduce the number of backorders without any additional investment in safety stocks. Conversely, with the current number of backorders, AFLC can reduce its investment in safety stocks. We illustrate by means of Figure 6.2. At present, the total number of annual backorders for the sample is about 10,000; the total safety stock is about $600,000. With the same investment in safety stock, we can immediately reduce our annual backorders to about 5750 units per year. This translates to a reduction of backorders for the total system of about (10,000 - 5750) X 200 = 850,000 units per year. Thus the number of backorders can be almost halved by incorporating leadtime variability and by adhering to the Presutti-Trepp optimal model.

7. LIMITATIONS, ASSUMPTIONS, RESEARCH QUESTIONS, METHODS AND ANALYSES USED

Limitations

The uninvolved reader may find fault with a study of this kind, but considering the limited scope originally envisaged, that reader may perhaps discern the amount of work that was nevertheless put in it. We are dealing here with inventories whose worth runs into the billions and whose management is necessarily very complex. Consequently, it would be vain to claim that this modest study will resolve all the problems of shortages encountered by AFLC.

Assumptions

Since we did not have daily demand, but only demand rates, it was convenient to assume that daily demand was Poisson-distributed. LORRA had supplied us with daily shipment data on two items, but shipment data are not demand data. The daily shipment data do not appear to conform to the Poisson distribution.

We had also assumed that leadtimes and demand rates were statistically independent. Lacking simultaneous demand and leadtime data, it was impossible to validate that assumption.

Finally, we implicitly assumed that the sample of fifty-six high-intensity items is representative of the population of such items. In fact, it was not a random but a convenience sample.

Research Questions

In a study of this kind, the questions raised by the research overwhelm the answers provided, and this study is no exception. Also the questions raised depend very much on the reader's point of view. From our point of view, there are numerous questions, and as an example we ask only two:

1. Why the incongruities in the mathematics of the current system? In calculating the safety factors, AFLC relies on a Presutti-Trepp formula which in turn is based on leadtime demand being Laplace-distributed. But in calculating the quarterly MAD, AFLC pretends that leadtime demand is normally distributed. One effect of this incongruity is in the calculation of the standard deviation of leadtime demand, σ, from MAD. We see that for the normal,

$$\hat{\sigma} = 1.25 \text{ MAD} \quad , \tag{7.1}$$

whereas for the Laplace,

$$\hat{\sigma} = \text{MAD} \quad . \tag{7.2}$$

Thus, with Laplace leadtime demand, we should use (7.2) rather than (7.1).

2. What is a proper definition of fill ratio? Does fill ratio address the percentage of requisitions filled or the annual fraction of demand satisfied? Using the latter as a definition of theoretical fill ratio does not produce results that are compatible with actual fill rates, as may be seen in Table 7.1. Of course, the calculation of theoretical fill ratio in Table 7.1 does not consider customer priority.

TABLE 7.1

CURRENT FILL RATES: REPORTED VS. THEORETICAL

		Fill Rates		
Item no.	FSN	Reported[1]	Theoretical[2]	$1 - \beta_T/Q$
9	5895004451349	81.90	99.0%	99.6
8	5895009497160	63.06	85.0	93.8
7	5960004009106	98.79	74.5	74.9
3	5962004537739	67.80	90.4	92.3
10	5895000894403	98.75	80.6	91.7
2	5841000738392	100.00	95.9	98.74
15	589001167508	73.29	82.4	93.33
11	5831008803563	99.76	69.0	67.4
13	5821009906461	100.00	100.0	100.0
12	5821002694508	87.28	91.2	98.2
50	7045008479020	99.41	98.6	99.7
51	3040006211345	17.09	93.3	98.2
52	1670007970253	72.57	100.0	100.0
53	1560009492087	95.08	81.3	87.6
28	1095008747369	100.00	90.7	93.1
27	1095009120243	95.92	99.9	100.0
29	1095008614744	100.00	99.0	99.8
30	1095005227703	100.00	98.7	99.6

TABLE 7.1 (Continued)

Item no.	FSN	Fill Rates Reported[1]	Theoretical[2]	$1 - \beta_T/Q$
31	1095105168069	98.49%	98.9%	99.7
32	1095004918487	96.47	99.7	99.9
33	1095004335657	77.70	94.0	99.3
34	1095001111640	84.73	79.9	87.6
35	1005008938230	95.55	85.3	94.0
36	1005007889718	99.25	100.0	100.0
37	1005007879802	93.55	97.6	83.6
38	1005007755579	98.96	98.6	99.4
39	1005007545293	100.00	98.9	99.7
40	1005007545267	100.00	96.0	99.4
41	1005007545266	100.00	96.9	99.1
42	1005007016793	100.00	91.2	97.2
43	1005006999923	100.00	89.7	94.5
44	1005006999931	100.00	97.9	99.4
45	1005006999882	100.00	91.0	92.2
46	1005000511579	96.15	100.0	100.0
47	1005003357318	100.00	89.4	90.5
48	1005000178809	100.00	100.0	100.0

[1]From fill rates for April 1979 - March 1980 obtained from LORRA

[2]From a computer simulation of the current system

Indeed, there are several methods of calculating theoretical fill ratios. Brown (1967, pp. 91-92) comments on this, suggesting to us the use of $1-\beta_T/Q$. This alternative computation is also given in Table 7.1.

Methods and Analyses Used

In fitting leadtime data to the exponential, the gamma, the normal, the Weibull, and the lognormal, we used an existing U.S. Army computer program (1971). In testing the leadtime data for randomness, we wrote our own program. We similarly wrote our own computer programs in simulating leadtime demand, fitting leadtime demand to the normal and the Laplace, doing ABC classification of inventory, and in simulating the current and optimal systems. Furthermore, we wrote a simulation program to show that the long right tail of the distribution of forecast errors produced by Demmy (1979) was due to a wrong statistical procedure.

8. FINDINGS AND CONCLUSIONS

The major finding of this study is that leadtime variability does indeed influence safety stocks, and in order to maintain these at appropriate levels, a one-time investment of about $150 million is necessary. Also necessary would be an increase in annual operating expenditure of approximately $50 million.

Even if AFLC does not wish to build up its stocks, it can immediately halve its number of backorders by explicitly incorporating leadtime variability in its calculations and by adhering to the Presutti-Trepp optimal model. This would require a different mix of items and, in general, larger order quantities.

We have also found it worthwhile to use an ABC classification of the high-intensity items. On the one hand, ABC classification is dynamic and thus much superior to the statitic SMGC in current use at AFLC. On the other, the ABC classification points directly to the few high-intensity items management should really concentrate its attention upon. In this regard the SMGC seems anachronistic.

APPENDIX A

MEMO OF UNDERSTANDING

(MARCH 7, 1980)

MEMO OF UNDERSTANDING

SUBJECT: Hayya Leadtime Variability Study - AF Contract F33615-79-C-5143

1. The results from Phase I of AF Contract F33615-79-C-5143, Leadtime Variability in Inventory Requirements Projection, were discussed in the 5 Feb meeting of contractor Dr. Jack C. Hayya, Capt Paul Gross/AFBRMC and representatives from LORRA. During the meeting it was determined that Phases II & III could not be conducted as originally planned. The leadtime distributions found in Phase I were for 16 specific items during the 76-79 time period. Thus the simulation model, INSSIM, which has 71-75 data cannot be used in Phase II.

2. Leadtime data on 46 more items have been given to Dr. Hayya and he has agreed to curve fit their distribution using the same methods as were used on the original 16 items. Also he will determine and analyze the convolution of the leadtime distributions with demand distributions assumed to be Normal, Laplace, or "skewed Normal". Dr. W. Steven Denny's results in "Statistical Characteristics of Forecasting Techniques for D062 Economic Order Quantity Items" shows distributions for demands that are near normal but skewed. The one in Table IV-2 for OC with SMGC=2 will be used for this study.

3. A data request for 76-79 demands for the 62 items on which Hayya has leadtime is being processed and should be available in March. Hayya will use this to determine the actual leadtime demands for the items and test the fit of the curves to the p.d.f.s specified in the original contract. This also will be compared to the p.d.f.s of the convoluted distributions discussed in paragraph two above.

4. Dr. Hayya has also agreed to investigate our Variable Safety Level formula. He will conduct an analysis of the sensitivity of the safety level to changes in variance from demand variance to leadtime demand variance. Typical holding costs range from .15 to .25. The implied shortage factor or Lagrangian multiplier normally ranges from 350 to 600. The Lagrangian multiplier is always adjusted so that each ALC will have a total current safety level stock of dollar value equivalent to 2 months supply of all stock at that ALC. Any indication of how the mix of items at an ALC will change when using leadtime demand variance is important. Dr. Hayya will also make recommendations on possible marginal analysis techniques which will use variable leadtime demands to minimize back orders.

5. A copy of "More Ado About Economic Order Quantities (EOQ)" by Victor J. Presutti Jr and Richard C. Trepp was furnished Dr. Hayya when he visited W-P AFB on 5 Feb 1980. Copies of the EOQ briefing, Vic Presutti's VSL briefing, and "Measurement and Implications of Production Leadtime Variability", by the US Army Inventory Research Office will be furnished Dr. Hayya along with a copy of this memo. Demand data on the 62 items will be furnished as soon as available.

APPENDIX B

EXPLANATION OF TABLE 6.1: OPTIMAL SYSTEM

This table can be sorted by item no. or by FSN. The table has 24 columns, 12 on page 1 and 12 on page 2. I shall use the data for item #48 to illustrate.

Col. 1: Item no., following the sequence in the Phase II report

Col. 2: FSN

Col. 3: Daily Demand, E(d)

Col. 4: Average leadtime in days, E(L)

Col. 5: Variance of leadtime, V(L)

Col. 6: Mean of leadtime Demand

$$\begin{aligned} E(x) &= E(L) \,.\, E(D) \\ &= (183.25)\,(21.70) \\ &= 3976 \,, \qquad \text{for item \#48} \end{aligned}$$

Col. 7: Variance of leadtime Demand

$$\begin{aligned} V^*(x) &= E(L) \,.\, V(D) + [E(D)]^2\, V(L) \\ &= E(L) \,.\, E(D) + [E(D)]^2\, V(L) \,, \text{ for Poisson demands} \\ &= E(x) + [E(D)]^2\, V(L) \\ &= 3976 + (21.70)^2\,(535) \\ &= 255{,}902 \,. \end{aligned}$$

(The computer result of 255,870 is more accurate)

Col. 8: The standard deviation of leadtime demand, $\sigma^*(x)$

$$\begin{aligned} \sigma^*(x) &= \sqrt{V^*(x)} \\ &= \sqrt{255870} \\ &= 505.84 \,. \end{aligned}$$

Col. 9: Unit cost, c_i

Col. 10: The Wilson Formula

$$Q_w = \sqrt{\frac{2A_iD_i}{a_ic_i}} \quad ,$$

where A_i = \$441.07, if c_iD_i > \$19,500, \$291.82 otherwise (See AFLCR 57-6 (C3), 22 June 1979),

D_i = Annual demand,

a_i = holding cost factor, which is 0.23 in this example,

c_i = unit cost .

Hence

$$Q_w = \sqrt{\frac{2(441.07)(650.96)(12)}{(0.23)(3.08)}}$$

$$= 3118.9 \ .$$

Col. 11: Optimal EOQ, Q^*, calculated according to formula (14) in Presutti and Trepp (1970, p. 249):

$$Q_i^* = 0.707\ \sigma_i^* + \left\{(2A_iD_i/a_ic_i) + \sigma_i^{*2}/2\right\}^{1/2}$$

$$= 0.707\ \sigma_i^* + \left\{Q_w{}^2 + \sigma_i{}^{*2}/2\right\}^{1/2}$$

$$= 0.707\ (505.84) + \left\{(3118.9)^2 + 255870/2\right\}^{1/2}$$

$$= 3497 \ .$$

What is necessary in order to use the above formula is for $\frac{Q_i{}^*}{\sigma_i{}^*} > 2$ or 3.

For item =48, $\frac{Q_i{}^*}{\sigma_i{}^*} = \frac{3497}{505.84} = 6.9$.

Col. 12: Optimal safety factor, k_i^*, calculated according to formula IV.2 in Presutti and Trepp (1970, p. 249):

$$k_i = -\frac{1}{\sqrt{2}} \ln \left[\frac{\sqrt{2} Q_i^* a_i c_i}{0.5(-\lambda) Z_i \sigma_i (1-\exp(-\sqrt{2}\ \frac{Q_i^*}{\sigma_i^*})} \right] .$$

Here we are using

$$-\lambda = 660 \quad , \quad Z_i = \frac{1}{2} \ , \ a_i = 0.23 \quad .$$

Then

$$k_i = -\ 0.707 \ \ln \left[\frac{\sqrt{2}(3497)(0.23)(3.08)}{0.5(660)(0.5)(505.84)(1-e^{-\sqrt{2}(3497)(505.84)}} \right]$$

$$= -\ 0.707 \ \ln \left[\frac{3503.39}{83{,}458.86} \right]$$

$$= -\ 0.707 \ \ln 0.04197$$

$$= 2.242 \ .$$

Col. 13: Safety stock $k^* \sigma^*$

$$k^* \sigma^* = (2.242))505.84)$$

$$= 1134 \qquad , \qquad \text{for item \#48} \ .$$

Col. 14: Value of safety stock,

$$c_i k^* \sigma^* = \$3.08\ (1134)$$

$$= \$3492 \ .$$

Col. 15: Service level. This is

$$1 - \frac{1}{2} e^{-\sqrt{2} k_i^{*}} = 1 - \frac{1}{2} e^{-\sqrt{2}\,(2.242)}$$

$$= 0.979$$

$$\approx 0.98 \ .$$

Col. 16: Weighted Backorders at any time, β_T. We may think of this as the number of requisitions backordered when $Z_i \neq 1$. If $Z_i = 1$, β_T would be the average number of units backordered at any time.

We use formula IV.2 (Presutti and Trepp, p. 249) or formula (10) (Presutti and Trepp, p. 246). From formula IV.2, we have

$$\beta_T = \frac{0.5}{2} \frac{Z_i \sigma_i^{*2}}{Q_i^{*}} \left(1 - e^{-\sqrt{2} \frac{Q_i^{*}}{\sigma_i^{*}}}\right) e^{-\sqrt{2}\, k_i^{*}} \ .$$

For item #48,

$$\beta_T = \frac{0.5}{2} \frac{(\frac{1}{2})(255870)}{3497} \left(1 - e^{-\sqrt{2} \frac{3497}{505.84}}\right) e^{-\sqrt{2}\,(0.242)}$$

$$= 0.38 \ .$$

The actual units backordered at any time would be

$$\frac{0.38}{Z_i} = \frac{0.38}{\frac{1}{2}} = 0.76 \ .$$

Col. 17: The fill ratio is

$$\frac{\text{Annual Demand - Annual backorders}}{\text{Annual Demand}}$$

Annual backorders are given in Col. 22. They are calculated according to formula (9) in Presutti and Trepp (1970, p. 246):

$$B_N = \frac{0.5}{\sqrt{2}} \frac{D_i \sigma_i^*}{Q_i^*} \left(1 - e^{-\sqrt{2}\, Q_i^*/\sigma_i^*}\right) e^{-\sqrt{2}\, k_i^*}$$

$$= \frac{0.5}{\sqrt{2}} \frac{(21.70 \times 360)(505.34)}{3497} (0.999943)(0.041976)$$

$$= 16.77 \text{ units per year .}$$

Actually,

$$B_N = \sqrt{2}\, \beta_T\, D_i / Z_i \sigma_i^*$$

$$= \sqrt{2}\, (0.38)(7812) \,/\, (\tfrac{1}{2})(505.84)$$

$$= 16.60 \text{ (the difference is due to rounding) .}$$

Hence, the fill ratio is

$$\frac{7812 - 16.77}{7812} = 0.998 \quad .$$

Col. 18: Average value of inventory on hand.

The formula is from Presutti and Trepp (1970, p. 247), where

$$\text{Value } E[OH] = (k\sigma + Q/2 + \beta_T)\, c_i$$

$$= (k\sigma + Q/2 + \beta_T/Z_i)\, c_i \ .$$

For item #48, this would be

$$\text{Value } [OH] = [1134 + \frac{3497}{2} + 2(0.38)]\ 3.08$$

$$= \$8880.44 \ .$$

Col. 19: Annual Ordering Cost. This is

$$\frac{A_i D_i}{Q_i} = (\$441)\ \frac{7812}{3497}$$

$$= \$985.15 \ .$$

Col. 20: Annual holding cost, based on average inventory position. This is

$$a_i c_i \ (\mu_i + k_i \sigma_i + \frac{Q_i}{2})$$

$$= 0.23(3.08)\ (3976 + 1134 + \frac{3497}{2}) = \$4858.56 \ .$$

Col. 21: Annual Operating Cost. This is from Formula IV.1 (Presutti and Trepp, p. 249):

$$\frac{A_i D_i}{Q_i} + a_i c_i \left(\mu_i + k_i \sigma_i + \frac{Q_i}{2}\right)$$

$$= 985.25 + 4858.80$$

$$= \$5844.05 \,.$$

Col. 22: Annual no. of backorders.

Col. 23: FSN

Col. 24: Item #

APPENDIX C

EXPLANATION OF TABLE 6.2: AFLC'S CURRENT SYSTEM

This table can also be sorted by item no. or by FSN. The table has 23 columns, 11 on page 1 and 12 on page 2. Again I use the data for item #48 to illustrate.

Col. 1: Item no.

Col. 2: FSN

Col. 3: Monthly demand, $E(D)$

Col. 4: Months leadtime, $E(L)$

Col. 5: Leadtime Demand:

$$E(x) = E(L) \,.\, E(D)$$

$$= (6.11)(650.96)$$

$$= 3977, \quad \text{for item no. } 48$$

Col. 6: Quarterly MAD

Col. 7: The std. deviation of leadtime demand, according to AFLCR 57-6(C2), 29 December 1978. It is

$$\sigma = 0.5945 \text{ MAD } (0.82375 + 0.42625 \text{ L})$$

$$= 0.5945\ (483.60) \left\{0.82375 + 0.42625\ (6.11)\right\}$$

$$= 985.59\ .$$

Col. 8: The EOQ. Using the EOQ table for WR-ALC (AFLCR 57-6(C3) 22 June 1979), we have

$$Q = 0.5D \quad , \quad \text{since } c_i D_i > \$10{,}000$$

$$= 0.5(650.96)(12)$$

$$= 3906 \text{ units} .$$

Col. 9: Unit cost

Col. 10: The safety factor, according to AFLCR 57-6(C2), 29 December 1978, p. 7-2. It is

$$K = -0.707 \ln \frac{2\sqrt{2}\, Q a_i c_i}{\lambda(\frac{1}{\sqrt{R}})\sigma(1-e^{-\sqrt{2}Q/\sigma})} \quad ,$$

where for our example

R = the average requisition size, 4 in this case,

λ = 660,

and the other terms as before. Hence

$$K = -0.707 \ln \frac{2\sqrt{2}(3906)(0.23)(3.08)}{660(\frac{1}{2})(985.39)\left(1-e^{-\sqrt{2}\,\frac{(3906)}{985.39}}\right)}$$

$$= -0.707 \ln \frac{7826.29}{323{,}983.2}$$

$$= -\ 0.707 \text{ in } 0.024156$$

$$= 2.63 \ .$$

The value of K must be between zero and three.

Col. 11: Safety Stock

$$k\sigma = 2.63\ (985.39)$$

$$= 2591.58 \quad , \quad \text{for item \#48} .$$

This value must not exceed that for leadtime demand.

Col. 12: \$ Safety Stock. This is

$$c_i K_i \sigma_i = 3.08\ (2594.27)$$

$$= \$7990.35 .$$

Col. 13: True K. The K-value of Col. 10 is not based on the true standard deviation of leadtime demand, σ^*. The value of σ^* obtained from Table 6.1, Col. 8 is 505.84 not 985.39. Hence the true K is

$$\text{True K} = \frac{\text{Safety Stock}}{\sigma^*}$$

$$= \frac{2594.27}{505.84}$$

$$= 5.129 .$$

Col. 14: Actual β_T weighted by essentiality, Z_i. To be consistent, we set

$$Z_i = \frac{1}{\sqrt{R}} = \frac{1}{2} .$$

Then using formula IV.2 in Presutti and Trepp (1970, p. 249), we have

$$\beta_T = \frac{0.5}{2} \frac{Z_i \sigma_i^{*2}}{Q_i} \left(1 - e^{-\sqrt{2} \frac{Q_i}{\sigma_i^*}}\right) e^{-\sqrt{2}\, k_i^*}$$

$$= \frac{0.5}{2} \frac{(\frac{1}{2})(505.84)^2}{3906} \left(1 - e^{-\sqrt{2}\left(\frac{3906}{505.84}\right)}\right) e^{-\sqrt{2}\,(5.129)}$$

$$= 0.006$$

The actual number of backorders expected at any time is actually

$$\beta_T/Z_i = 0.006/0.5 = 0.012 \ .$$

Col. 15: Actual Service Level

$$1 - \frac{1}{2} e^{-\sqrt{2}\, k_i^*}$$

$$= 1 - (0.5)\ (0.00070769)$$

$$\simeq 1.00 \ .$$

Col. 16: \$ Inventory on hand. This is

$$c_i \ (\text{Safety stock} + \frac{Q}{2} + \beta_T/Z_i)$$

$$= 3.08\ (2594.27 + \frac{3906}{2} + 0.012)$$

$$= \$14{,}005.$$

Col. 17: Fill Ratio. This is

$$\frac{\text{Annual Demand - Annual Number of backorders}}{\text{Annual Demand}}$$

We obtain the annual number of backorders using formula (9) in Presutti-Trepp (1970, p. 246):

$$\beta_N = \frac{0.5}{\sqrt{2}} \frac{D_i \sigma_i^*}{Q} \left(1 - e^{-\sqrt{2}\, Q/\sigma^*}\right) \left(e^{-\sqrt{2}\, k^*}\right)$$

$$= \frac{0.5}{\sqrt{2}} \ \frac{(12)(650.96)(505.84)}{3906} \ (0.99998)\ (0.00070769)$$

$$= 0.25 \ .$$

You see this value in Column 21.

Hence,

$$\text{Fill Ratio} = \frac{12(650.96) - 0.25}{12(650.96)}$$

$$\simeq 1.00 \ .$$

Col. 18: Annual Ordering Cost

$$\frac{A_i D_i}{Q_i} = \frac{441.07(650.96)(12)}{3906}$$

$$= \$882.09 \simeq \$882.14 \ .$$

It is strange that the ordering cost for so many items is $882.14 or $583.64. Apparently an annual ordering cost of $882.14 corresponds to an ordering cost of $441.07, whereas an annual ordering cost of $583.74 corresponds to an ordering cost of $291.82. Thus

$$\frac{\$291.82}{441.07} (\$882.14) = \$583.64 \ .$$

Consider item #3 where the dollar value of annual demands is 373.92 X (12)($2.74) = $12,294.49 < $19,500. Hence, the ordering cost, A = $291.82; and the annual ordering cost would be

$$\frac{AD}{Q} = \frac{(291.82)(373.92)(12)}{2244}$$

$$= \$583.52 \ .$$

It seems that the EOQ table for WR-ALC (AFLCR 57-6(C3), 22 June 1979, p. 7-14-1) is designed to produce precisely these annual order costs.

Col. 19: Annual Holding Cost based on the Inventory Position. This is

$$a_i c_i \left(\mu + \text{Safety Stock} + \frac{Q}{2}\right)$$

$$= \$0.23(3.08)\left(3976 + 2594.27 + \frac{3906}{2}\right)$$

$$= \$6037.88 .$$

Col. 20: Annual operating cost. This is the sum of the annual ordering cost and the annual holding cost.

Col. 21: Annual number of units backordered. This has already been calculated under Col. 17.

Col. 22: FSN

Col. 23: Item no.

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